Strategic Negotiations and Outside Options

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Abstract

In this paper we analyse how the economic literature has introduced in the negotiations the possibility of taking an outside option by any negotiator. Moreover, we study the robustness of the Outside Option Principle, which means that only credible outside options affect the outcome of any negotiation.

1) Introduction

In general terms, bargaining is the process of arriving at mutual agreement between two or more individuals. The most common bargaining problem consists of "dividing the dollar" between two players. In the economic life this situation is typical of negotiations between a trade union and a firm, a buyer and a seller, etc.

In this context, there is a broad agreement about the fact that in real negotiations either player can usually quit negotiation taking up the best option available elsewhere. This best option is known as outside option.

This paper analyzes in which ways the economic literature has incorporated outside options into a negotiation and, also, how the bargaining outcome is affected by the presence of this new element.

Negotiations have been studied traditionally by the cooperative bargaining theory, where assumptions are made about the bargaining solution without specifying the bargaining process itself. In this approach, an outside option payoff amounts to a *status quo* point. Namely, when a player has an outside option her bargaining power is automatically increased.

Nevertheless, a new approach arised in the eighties: the strategic approach, where the outcome is an equilibrium of an explicit model of the bargaining process. In this approach, a strategic bargaining game is used for introducing outside options in a negotiation. In particular, in every case, the sequential bargaining game in extensive form formulated by Rubinstein (1982) is taken as a reference model. In Rubinstein's model two players make in turn proposals as how to divide a pie, assuming that all the relevant elements of the negotiation are common information for both players.

The strategic approach and, in particular, Rubinstein's model are more suitable for the introduction of outside options mainly by two reasons:

(i) It clearly divides the bargaining problem in two elements: the players' preferences and the rules of the bargaining game. If we assume that the

[1] I would like to thank Gonzalo Olcina for their useful comments which greatly improved this paper.

preferences are constant, we can study in which way a modification in the rules of the game alters the bargaining outcome. Specifically, one modification might be the possibility of opting out for any player.

(ii) If we require that players' strategies constitute a Subgame Perfect Equilibrium, Rubinstein's model will have a unique and, thus, efficient equilibrium. These two properties of Rubinstein's equilibrium are very important for the normative economics. Therefore, if the incorporation of an outside option into the Rubinstein's model moves the bargaining outcome away from the uniqueness and efficiency above mentioned, we can conclude that the outside option has no good effects normatively.

We carry out our analysis of the role of the outside options in negotiations within the framework of Rubinstein's model because, given the reasons mentioned before, can be more useful to isolate the strategic effects of the outside options. Hence, we begin our paper with a brief description of the Rubinstein bargaining game given that through the paper we modify this model by allowing the players to opt out at differents moments of the game.

In Rubinstein's model, we need to apply the Subgame Perfect Equilibrium (SPE) notion because the Nash equilibrium concept puts very few restrictions on the possible outcomes of the game. Then, applying the SPE notion we have a unique and, thus, efficient solution which, under some conditions coincides with the Nash Cooperative Solution, known as split-the-difference outcome.

This paper is divided in two main parts. In the first part, assuming fixed and exogenous outside options, we impose to the players different restrictions to take their outside options. The objective of such analysis is to study if deviations from equilibrium payoffs and equilibrium properties of the Rubinstein's model depend on players' restrictions to opt out.

We begin studying Ponsatí and Sákovics (1995)'s model, where players can opt out freely, that is, any player is allowed to opt out in any period whenever an offer is rejected. Hence, after the rejection of a proposal both the proposer and the responder can opt out. However, the proposer has a large bargaining power because her proposal will be a take-it-or-leave-it offer. Hence, if her threat to opt out is credible she can give the responder her outside option payoff and appropiate the rest of the pie. Consequently, the responder will have a very little bargaining power because she can get at most her outside option payoff.

The fact that both parties can opt out as proposers will be the key point of the model. Given that today's proposer will be tomorrow's responder she knows that the large bargaining power of today will be transformed into a little bargaining power tomorrow. Thus, any proposer's threat to opt out will be credible regardless of the outside option value, which in turn provokes that Rubinstein's equilibrium strategies are altered even for zero-sized outside options.

This is a striking finding because contradicts a classical result settled by the literature: the Outside Option Principle (O.O.P). The O.O.P establishes that only the outside options with payoffs superior to the equilibrium payoffs obtained in the bargaining game without outside options, affect the equilibrium strategies of the game.

The O.O.P has become so important partly because contradicts clearly the results of the Cooperative bargaining theory, that is, it yields in some cases an equilibrium payoff different from the split-the-difference outcome. But in our case, although the O.O.P is not held, the bargaining outcome will be different from the split-the-difference solution because we will have a multiplicity of equilibria and, thus, an indeterminacy in the resultant equilibrium (except for high-sized outside options).

The reasons for the existence of multiplicity of equilibria have to be looked for on the possibility of opting out as proposer for some player. This is what we explain in the next model from Shaked (1994).

In Shaked (1994)'s model we assume that only a player can opt out as a proposer. As a result, the O.O.P holds in this context. This is so because the player who has an outside option as proposer, given that her rival can't opt out as proposer, can obtain a good payoff even as responder. Hence, opting out will not be a credible threat for some low-sized outside options of this player.

However, likewise in Ponsatí and Sákovics (1995)'s, in Shaked (1994)'s there is a multiplicity of equilibria for some outside options values. The reasons for the indeterminacy of equilibria are that in both models at least a player can opt out as proposer. When a proposer has an outside option, in order to decide if it is convenient its execution, she compares her opting out payoff with the payoff that she will get the next period inside the negotiation (continuation subgame payoff). The problem is that it arises a large gap between the best and the worst payoff that she can obtain in the continuation subgame. In the best payoff the proposer has a credible threat to opt out through the continuation subgame and, thus, can get the whole cake in Shaked (1994) or the part of the pie that survive after providing the responder with her outside option payoff in Ponsatí and Sákovics (1995)'s. But in the best payoff the outside option is not credible and, thus, this player obtains the Rubinstein's payoff. Consequently, if the proposer's outside option payoff is situated between her best and her worst payoff of the continuation subgame, then the notion of SPE cannot restrict the set of equilibria because we will find a different equilibrium for every different players' conjecture about the continuation subgame payoff of the proposer.

The above result is verified in the next model: a modified version of Shaked and Sutton (1984)'s. In this model we allow both players to opt out but only as responders, that is, the bargainers are locked as proposers. In this case, the O.O.P holds too. But contrary to the previous models the equilibrium is unique and, thus, efficient. Notice that when a responder has an outside option, the proposer, given that she can make the last offer before the responder opts out, can ensure that her rival not obtain more than her outside option payoff. Therefore, there will not be a gap between the best and the worst continuation subgame payoffs and, then the notion of SPE yields a unique equilibrium.

In the second part of this paper, we move forward one step quitting the assumption of fixed and exogenous outside options. About this topic the literature is just emerging because until very recently it was thought that once settled the Outside Option Principle the role of the outside options was completely established and there was no necessity in adding new unnecessary details that only would yield more entangled models.

However, if we think over this issue one can question the validity of assuming an invariable and certain opting out payoff in a real world so dynamic and changing. Furthermore, one can logically conclude that many relevant elements for the bargaining outcome are missed.

Our main goal in this part of the paper is to show through three simple bargaining models that new issues arise in the determinacy of the equilibria when we face more complex outside options.

We, thus, begin considering a model formulated by Rubio (1994) where we assume that the outside option is a function dependent on time. Although this function is given exogenously to the model, this "new" outside option is incorporated to a model where only the responders can opt out and, as a result, the O.O.P is not only held but also is strengthen. In this context, the outside options take a different value in every period, then we just need that in one period the outside option payoff is greater than the Rubinstein's payoff to find that the bargaining outcome is affected by the outside option. Therefore, the resultant equilibrium will depend on the concrete functional form that the outside option takes.

Next, we proceed to consider endogenous outside options. In particular, we study a variant of Compte and Jehiel (1997)'s model where the outside options payoffs depend on what the players do during the negotiation, that is, we have history-dependent outside options. In this case, it's possible to find inefficient equilibrium. When a player makes an offer, she increases the outside option of her rival and, thus, the other player has incentives to opt out right away. In order to avoid this incentives, players will begin the negotiation making non-serious offers until they reach an agreement in period n. However, in this model it is assumed that the players face a cost when they opt out. If this costs are low enough we can have that the inefficiency of the delayed agreement is higher than the inefficiency of opting out. Thus, in this situation players will prefer to opt out right away rather than to reach the delayed agreement.

Finally, we examine Ponsatí and Sákovics (1996)'s model where it exists uncertainty on the sizes of the outside options. It is assumed the existence of three crucial periods in the life of any outside option : a first period in which the outside options are available, a uncertainty revelation period and a maturity period. All the three periods will be important in the determinacy of the equilibria but the uncertainty revelation period will be the most crucial, that is, when the uncertainty is solved will determine the resultant equilibrium. For instance, depending on the distance of the uncertainty revelation period from the beginning of the negotiation and the parameters, we can face a unique and inefficient equilibrium or an equilibrium where the players take their outside options before their value is known.

In the next sections we analyze in detail the role of the outside options in a bargaining game. In section 2 we present a short version of the Rubinstein's alternating offer bargaining game. In section 3 we study three bargaining models where the players have different restrictions to opt out, assuming that the outside options payoffs are fixed and exogenous to the models. First, we examine a model where both players can quit negotiation whenever they want. Then, we describe a negotiation where a player is locked as proposer and finally, we consider a case where players can't opt out as proposers. In section 4 we relax the assumption of fixed and exogenous outside options and analyse three more complex outside options. We begin with a time-dependent outside option. Then, we study an endogenous history-dependent one and finally we examine a bargaining model where players' opting out payoff is uncertain. In section 5 we conclude with general remarks and further research areas.

2) <u>Rubinstein's alternating offer bargaining game</u>

Let us first describe briefly Rubinstein's model because it is a reference point in almost every bargaining game with outside options.

Suppose two players that are bargaining over a "pie" of size 1. The players alternate in making offers, starting with player 1. An offer *a* made in period t can be either accepted or rejected by the other player. If it is accepted, the bargaining ends and the agreement *a* is implemented. If the offer is rejected, the play passes to period t+1 and the player who has rejected makes a counteroffer. The game continues in this manner until an agreement is achieved.

Players' preferences depend on not only the shares of the pie they receive, but also the period of the agreement. Specifically, we will assume to simplify the following utility function: $U_i(a_i,t)=\delta^t a_i$. This utility function implies stationary preferences and stationarity will simplify the resolution of the model because all subgames in which a given player makes the first offer will have the same strategic structure.

If we examine the possible equilibria of the game, unfortunately the Nash equilibrium notion is not useful because it puts very few restrictions on the possible outcomes of the game. This is so because it doesn't exclude non-credible threats. Thus, we need a stronger notion of equilibrium : the Subgame Perfect Equilibrium (SPE), where players' strategies constitute a Nash Equilibrium after every possible history of the game.

In any subgame H, we can derive two equations applying the notion of SPE. In this equations, any player (in equilibrium) must be indifferent between the payoff of accepting her rival's offer and the payoff of her next period own offer. Solving them, we obtain an immediate agreement and, thus, efficient where the players' SPE payoffs are : $\left(\frac{1}{1+d},\frac{d}{1+d}\right)$. We can also show that this SPE is unique using Shaked and Sutton (1984) analysis. These authors utilises the stationarity and symmetry of the game to derive bounds for the supremum (*M*) and the infimum (*m*) over the set of SPE payoffs for the proposer in period t. Because the responder will make an offer in the next period t+1, proposer's reservation value in period t is between these two payoffs [δm,δM]. That is, we can find the next bounds for m and M :

$$M \leq 1-\delta m$$
$$m \geq 1-\delta M$$

If we take into account the constraint $M \geq m$, these equations are solved by the unique solution :

$$M=m=\frac{1}{1+d}$$

Rubinstein's solution is a reference point in most of non-cooperative bargaining games because, apart from its uniqueness and efficiency, it coincides with Nash cooperative solution (when $\delta \rightarrow 1$ or the lengths of periods approaches to 0), known as split-the-difference outcome.

Nevertheless, Rubinstein's model assumes implicitly that bargainers are "locked" inside the negotiation, that is, they are restricted to either agreeing in any period or achieving the perpetual disagreement. Through this paper, we will relax the former assumption allowing players to leave the negotiation when they take an outside option. On the one hand, we show in which way outside options can alter the outcome of "simple" negotiations. On the other hand, we analyse if this changes just alter equilibrium payoffs or can also affect to equilibrium properties (inefficiencies, multiplicity). Furthermore, we capture some mechanisms joined to outside options which play an important role in the determinacy of the equilibria.

3) <u>Is it so important who has the possibility of opting out ?</u>

The aim of this section is to show that the changes that provoke an outside option on a bargaining game depends crucially on if one or both players have the possibility of opting out and, also, if they can take their outside option either as proposers or as responders.

In this sense, we will demonstrate that the role of outside options is more complex than what was established at first by the literature. When Binmore et al (1989) settled the Outside Option Principle (O.O.P), mainstream literature ended the discussion about outside options because it was believed that this principle captured exactly their effects. The O.O.P establishes that only those outside options which payoffs are superior to Rubinstein's equilibrium payoffs have any effect on equilibrium strategies, and it is so important because yields in some cases an equilibrium payoff different from split-the-difference outcome.

Nevertheless, we will show that O.O.P validity[2] will depend crucially on players' restrictions of opting out, that is, the O.O.P will not be valid when *both* players can quit negotiation whenever they want. Furthermore, we will see that outside options apart from changing equilibrium payoffs can also alter equilibrium properties. For instance, in some cases there will exist multiplicity of equilibria and, thus, inefficiencies.

We analyse three cases where the players have different restrictions to take their outside option. In particular, we begin with a case where both players can quit negotiation whenever they want. Then, we study a negotiation where a player is locked as proposer and finally, we examine a case where players can't opt out as proposers. As we will see now, both equilibrium payoffs and equilibrium properties are seriously affected by these restrictions.

3.1) <u>Opting out freely</u>

In Ponsatí and Sákovics (1995)'s model it is shown that when we don't impose any restriction on players' opportunities to take up their outside options, the results of a bargaining game are altered dramatically.

[2] It can also be shown that O.O.P is not valid when Rubinstein's model with outside options is modified by either incorporating players that don't discount payoffs but face a probability of breakdown after every rejection (Binmore, Rubinstein and Wolinsky, 1986) or introducing a decaying factor in the size of the cake (Dalmazzo, 1992).

In particular, these authors assuming that any player can take their outside option in any period break the uniqueness of Rubinstein's equilibrium. Moreover, The O.O.P will not be valid because even outside options of size zero will alter the bargaining outcome.

Let us now proceed to describe the model in some detail. Suppose Rubinstein's bargaining game modified by allowing any player to opt out whenever an offer is rejected in any period. If either of the two parties take their outside option, both of them obtain respectively their outside option payoff (d_1,d_2). We assume that both players have the same constant discount factor $0 < \delta < 1$[3]. Then, if they reach the agreement *a* in period t, player's payoffs are respectively ($\delta^t a_1$,$\delta^t a_2$). Similarly, if either of the players takes her outside option in period *s* their payoffs are ($\delta^s d_1$,$\delta^s d_2$). Furthermore, given that the sum of the two outside options payoff is inefficient, $d_1+d_2 \leq 1$, players benefit mutually from trade.

An interesting feature of this model is that in any period either the proposer or the responder can opt out. But the question is : Is there any difference between opting out as a proposer and as a responder ?.

The answer is yes. Notice that when a proposer has the possibility of opting out, it appears an ultimatum minigame because when a proposer makes an offer, it is a take-it-or-leave-it offer. In this sense, the proposer threats the responder with taking her outside option if she rejects her offer. If the threat is credible, the proposer can appropriate the entire surplus. This threat will be credible when proposer's opting out payoff be larger than proposer's continuation subgame payoff. Hence, opting out as a proposer gives this party the possibility of appropriating the entire surplus[4] whenever taking the outside option is a credible threat.

Nevertheless, when a responder has the possibility of opting out and it is credible, the outside option just raises her reservation value payoff. This is so because the proposer has the right to make the last offer. Hence, when responder's

[3] Ponsatí and Sákovics (1995) assume that every player has a different discount factor, $\delta_1 \neq \delta_2$.
[4] As we will see soon, this is not exact in this model. Given that the responder can opting out in the same period to the proposer, she will not accept any payoff inferior to her outside option payoff, d_j. Therefore, we are referring to $1-d_j$ when we say that the proposer appropriates the entire surplus.

threat of opting out is credible, she can ensure that the responder not obtain more than her outside option payoff.

The fact that both parties can opt out as proposers is the key point of the model, because today's proposer will be tomorrow's responder she knows that today has a large bargaining power which will be transformed into a little bargaining power tomorrow. Thus, any proposer's threat of opting out will be credible. This is the reason because proposer's outside option payoff is always larger than worst continuation subgame payoff : if player i is today's proposer she can appropriate the entire surplus today (minus responder's outside option payoff) but in next period player j will be the proposer, so tomorrow player j will be who appropriates the entire surplus. Any player i's outside option, whichever value, will be larger than tomorrow's continuation value where player i is the responder.

Given that proposer's opting out payoff is always larger than her worst continuation subgame payoff, there are some players' beliefs for which proposers' opting out threat is credible. Therefore, Rubinstein's equilibrium strategies are altered in any case. In order to understand latter statement, notice that if proposer's outside options payoff had been inferior to worst continuation payoff, there would not exist any players' beliefs for which proposer's outside option threat would be credible. As a result, players would not take into account the outside options and we would have Rubinstein's unique equilibrium.

The bargaining outcome will depend on the size of the outside options. If they are large enough to exceed best continuation subgame payoff, the bargaining outcome will be a unique equilibrium. If, to the contrary, they are between the worst and the best continuation payoff we will find multiple equilibria.

We can distinguish between the following two cases :

i) *Large enough-sized outside options* ($d_1 > \delta(1-d_2)$ **or** $d_2 > \delta(1-d_1)$):

Firstly, on the one hand notice that in any subgame player i's payoff is never inferior to d_i because it is her opting out payoff. On the other hand, player i's maximum payoff is $(1–d_j)$[5] because player j will reject any payoff inferior to d_j.

Assume that the game starts with player 1 offering. In this case, player 1's threat of opting out will be credible because her outside option payoff is higher than the best continuation subgame payoff ($d_1 > \delta(1–d_2)$). Therefore, her take-it-or-leave-it offer is credible and the unique SPE will be given by the agreement $(1–d_2,d_2)$, where player 2 gets her reservation value payoff. Moreover, The former agreement will be the unique equilibrium even when player 1's opting out threat not be credible $(d_1 \leq \delta(1–d_2))$ if player 2's outside option payoff is larger than her best continuation payoff $(d_2 > \delta(1–d_1))$, because in this case player 2 will prefer (cheerlessly) to take her outside option before continuing bargaining to next period.

ii) *Not large enough-sized outside options* $(d_1 \leq \delta(1–d_2)$ **and** $d_2 \leq \delta(1–d_1))$:

Firstly, notice that outside option payoffs are always higher than the worst continuation payoff ($d_i > \delta d_i$, for any $0 < \delta < 1$). Therefore, we can't say that outside options are *never* a credible threat in this model.

Secondly, given that $d_i \leq \delta(1–d_j)$, outside option payoffs will be always lower than the best continuation payoff. Therefore, outside option payoffs will be situated between the best and the worst continuation subgame payoffs (that is, between the best and the worst player's possible world). There will exist many possible combinations and, thus, multiple equilibria which can be supported by strategies where any player's deviation is punished by a switch to this player's worst possible world (her worst continuation payoff). The resulting equilibrium will depend on players' beliefs about continuation subgame payoff.

Player 1's best equilibrium is achieved when player 2 is pessimistic about her continuation subgame payoff. Given player 2's pessimism, player 1 offers her to get her opting out payoff (d_2) which is accepted by player 2. In this case,

[5] Given that $d_i+d_j \leq 1$, we always have that $d_i \leq 1–d_j$.

equilibrium partition $(1–d_2,d_2)$ coincides with the former case equilibrium partition.

Player 1's worst equilibrium is attained when player 2 is optimistic about her continuation subgame payoff. Player 2 believes that she will get $\delta(1–d_1)$ in the continuation subgame. Player 1, knowing player 2's beliefs and given that $d_2 \leq \delta(1–d_1)$, offers $(1–\delta(1–d_1),\delta(1–d_1))$ which is accepted by player 2.

Between $[1–\delta(1–d_1),1–d_2]$ (the two extreme equilibria), any player 1's payoff can be supported in equilibrium.

<u>Theorem</u> [Ponsatí and Sákovics, 1995]

i) *If either $d_1 > d(1-d_2)$ or $d_2 > d(1-d_1)$, then we have a unique equilibrium given by the partition $(1–d_2,d_2)$.*

ii) *If both d_1 £ $d(1-d_2)$ and d_2 £ $d(1-d_1)$, then there exists a multiplicity of equilibria. In particular, for every q Î $[1-d(1-d_1),1-d_2]$ there is a SPE that ends with immediate agreement on $(q,1–q)$[6].*

The Outside Option Principle states that the bargaining outcome is not affected by non credible outside options. This principle has been widely accepted by the literature, despite the fact that many experiments deny it. In this sense, we have seen in this section that the validity of the O.O.P will depend crucially on players' freedom of opting out : when both players can quit negotiation whenever they want taking up their outside options, then the O.O.P looses its validity. Moreover, even zero-sized outside options $(d_1=d_2=0)$ affect the bargaining equilibrium. In this case, player 1's equilibrium payoffs will be within $\theta \in [1–\delta,1]$. Thus, there will exist a multiplicity of equilibria.

The reason for the O.O.P invalidity has to be looked for on the possibility of opting out as proposers for both players.

<u>Corollary 1</u>

[6] It is also possible to find delayed agreements.

When in a negotiation <u>*both*</u> *players have the possibility of opting out as proposers, then the O.O.P looses its validity.*

Corollary 1 points out a requisite to be hold in a negotiation necessary to O.O.P validity : At least one player has to be locked inside the negotiation as proposer. In the next section, we will illustrate better this point assuming that a player, player 1, can't opt out as proposer.

3.2) **<u>When a player is "locked in" as proposer</u>**

Let us assume as Shaked (1994) that only player 2 can opt out as proposer. Furthermore, in order to simplify suppose that neither player can take their outside option as responder[7].

The bargaining procedure will be identical to the one from previous section with the difference that only player 2 can opt out but only when she is the proposer of the game. In case player 2 takes her outside option in period t, the negotiation will end and both players will obtain respectively $(0,\delta^t d)$.

The main point here is player 2's outside option credibility. Given that she can opt out as proposer, when the threat of opting out is credible player 2 will get the entire "pie".

We can distinguish three different cases depending on the outside option value :

(i) When the outside option payoff is lower than the worst continuation payoff $\left(d < \frac{d^2}{1+d}\right)$, then the players don't take into account player 2's outside option. Thus, here the equilibrium coincides with Rubinstein equilibrium.

The intuition behind this result is that when player 2's outside option payoff is inferior to the worst continuation payoff, then her outside option is not a

[7] The main goal of this section is to show that when only a player can opt out as proposer the properties of the bargaining outcome change respect to the case when both players can opt out as proposers. Given that in the previous section we have seen that responders' outside options just modify their reservation value payoff, we can delete them without altering equilibrium properties.

credible threat because its payoff is lower than the payoff that she would get inside the negotiation. Moreover, notice that this result gives validity to the O.O.P.

Remember that this result didn't appear in the previous section because there players' outside option payoff were always greater than the worst continuation payoff.

(ii) When player 2's outside option payoff is greater than the best continuation payoff ($\delta^2 < d < 1$), then player 2's unique optimal strategy is to take up her outside option. In this case, the opting out payoff is so high that player 2's best action is to opt out.

In order to illustrate which is the best continuation payoff, let us assume that player 2 can opt out in period t. Player 2's best payoff in period t+1 will arise when opting out is always a credible threat in the subgame that begins in t+1. Given that player 2 only can opt out as proposer, she has to wait until period t+2 for threatening with opting out and, thus, for obtaining the whole cake (0,1) through her take-it-or-leave-it offer. Going backwards from t+2 until t, we obtain that the maximum payoff player 2 can get is δ^2. Therefore, δ^2 will be the best continuation payoff of player 2.

So, when $d > \delta^2$ player 2' threat of opting out will always be credible. Given that until the second period player 2 won't be permitted to opt out, player 1, knowing that player 2 will offer (0,1) in the second period and that she will have to accept it because player 2's outside option is credible, will take advantage of the reduction on the size of the cake to offer the agreement ($1-\delta,\delta$) in the first period. This offer will be the unique equilibrium partition.

(iii) When player 2's outside option payoff is situated between the best and the worst continuation payoff of player 2 $\left(\frac{d^2}{1+d} \leq d \leq \delta^2\right)$, then there will exist multiple equilibria which are sustained by a pair of strategies that punishes any players' deviation from equilibrium strategies with the worst possible equilibrium for this player. The extreme points of these interval of equilibria are the two former cases.

The best player 2's payoff from the interval of equilibria coincides with player 2's payoff from case (ii), where the outside option had a high size. In this equilibrium, player 2's threat of opting out is always credible and she gets the payoff δ.

On the other hand, the worst player 2's payoff from the interval of equilibria will arise when player 2's threat of opting out is never credible. Let us illustrate in more detail this point assuming that player 2 can take her outside option in period t. To continue bargaining in period t+1, player 1 has to offer her at least a payoff x equal to d. Given that x is obtained in the following period : $\delta x \geq d \rightarrow x \geq \frac{d}{d} \rightarrow x = \frac{d}{d}$. Namely, $\frac{d}{d}$ is the worst payoff that player 2 can get from the interval of equilibria[8] .

Between the two extremes, any player 2's payoff belonging to $[\frac{d}{d}, \delta]$ can be sustained as a SPE by the players' optimal strategies. These strategies punish any player's deviation by a switch to the extreme equilibrium in which this player obtain her lowest payoff.

<u>Proposition</u> [Shaked,1994]

Consider Rubinstein's alternating offer model, in which player 2 can opt out only as proposer.

Then :

(i) If $d < \frac{d^2}{1+d}$*, then the game has a unique SPE which coincides with Rubinstein's unique SPE.*

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[8] Notice that when d achieves the inferior bound from the interval $\frac{d^2}{1+d} \leq d \leq \delta^2$, player 2 will get $\frac{d}{d} = \frac{d}{1+d}$, which coincides with the case (i) of low-sized outside options.

(ii) If $\frac{d^{2}}{1+d}$ *£ d £ d^2, then there are multiple equilibria. Specifically, for every*

c Î [1 - d,1 - $\frac{d}{d}$*] there is a SPE that ends with the immediate agreement (c,1 - c).*

In every SPE player 2' s payoff is at least $\frac{d}{1+d}$*. The pair of strategies that generates every SPE from the interval c is given in Table 1.*

(iii) If d^2 < d < 1, there is a unique SPE that ends with the immediate agreement (1–d,d).

		z^*	d/δ	EXIT
	Proposes	$(1-z^*,z^*)$	(1–d/δ,d/δ)	(1–δ,δ)
Player 1	Accepts	$b_1 \geq \delta(1-z^*)$	$b_1 \geq \delta(1-d/\delta)$	$b_1 \geq 0$
Player 2	Proposes	$(\delta(1-z^*),1-\delta(1-z^*))$	(δ(1–d/δ),1–δ(1–d/δ))	(0,1)
Player 2	Accepts	$a_2 \geq z^*$	$a_2 \geq d/\delta$	$a_2 \geq \delta$
Player 2	Opt Out	NO	NO	YES
	Transition	Go to EXIT if player 1 proposes *a* with $a_1 > 1-z^*$	Go to EXIT if player 1 proposes *a* with $a_1 > 1-d/\delta$	Go to d/δ if player 2 continues bargaining after player 1 rejects a proposal.

Table 1

Through this model, we have shown that when there is only a player who can opt out as proposer the O.O.P is valid, contrary to the previous model where both players could take their outside options as proposers. Remember that when both players could opt out as proposers, a player has so little bargaining power as responder that when this player is the proposer of the game her threat of opting out is credible at least for some players' beliefs and, thus, players' equilibrium strategies are altered. However, in this model given that only player 2 can opt out as proposer, she can obtain a good payoff even as responder. So, opting out will not be credible in any case for some low-sized outside options of player 2.

Therefore, the difference between a negotiation where both players can opt out as proposers and a negotiation where only a player can opt out as proposer is the validity of the O.O.P. However, there is an important coincidence between them : a multiplicity of equilibria and, thus, the existence of delayed agreements.

Multiple equilibria will arise when either of the players is permitted to opt out as proposer.

Let us explain this latter point in some detail. Suppose the model of the present section where player 2 can opt out as proposer. On the one hand, when player 2's threat of opting out is credible, she gets the whole cake. On the other hand, when player 2's threat of opting out is not credible, her payoff coincides with Rubinstein's payoff. Therefore, player 2's outside option opens a "huge" gap between the best continuation subgame payoff of player 2 (which will be obtained when player 2's threat of opting out be always credible and, thus, in the first period where player 2 is permitted to opt out, she will appropriate the entire cake) and her worst continuation subgame payoff (which will be obtained when player 2's threat of opting out be never credible and, thus, she will get Rubinstein's payoff). If player 2's outside option payoff is situated between her best and her worst continuation subgame payoff, then the notion of SPE yields many equilibria because we will find a different equilibrium for every different players' conjecture about the continuation subgame payoff of player 2[9].

Corollary 2

When in a negotiation at least a player has the opportunity of opting out as proposer, then there will be a multiplicity of equilibria for some outside options values.

From Lemma 2 we conclude the importance of opting out as proposer because when any player has this possibility, the outside options apart from altering players' equilibrium payoff can modify equilibrium properties like the uniqueness of the equilibrium.

In the following section, we show that when players can't opt out as proposers the multiplicity of equilibria disappears, while the O.O.P is still valid.

3.3) Uniqueness and opting out as responder

[9] A similar intuition can be applied to Ponsatí and Sákovics (1995)'s model.

Shaked and Sutton (1984) presents a bargaining model in which a player could opt out as responder. In this section, we will modify slightly this model allowing both players to opt out as responders. This modification just increases reservation value payoff of the player who didn't possess an outside option in Shaked and Sutton (1984)'s model and will not affect to the equilibrium properties.

Assume, then, that both players can unilaterally quit the negotiation by taking their outside option but only when they are the responders of the game. Therefore, if player 1 opts out in period t then the players obtain the utility pair $(\delta^t d_1,0)$. On the other hand, if player 2 opts out in period t+1 players obtain $(0,\delta^{t+1} d_2)$.

As we explained in Ponsatí and Sákovics (1995), when player i has an outside option as responder which is credible, player i' s outside option just raises her equilibrium payoff from $\frac{d}{1+d}$ to d_i (her outside option payoff). This is so because player j, as the proposer of the game, can make the last offer before player i opts out, so she can ensure that player i not obtain more than d_i.

Again, player i' s outside option will be credible when the outside option payoff is greater than the continuation subgame payoff. But now, the credibility of player i' s outside option will depend on player j's outside option because the continuation payoff of player i relies on the credibility of player j' s outside option.

Given that the negotiation begins with player 1 as proposer, player 2 will have the possibility of opting out as responder in the first period. If both players' outside options are not credible, we will find again the Rubinstein's equilibrium. If only player 1's outside option is credible, given that it will not be executed until period 2, the resultant equilibrium is solved going backwards from the partition of the second period $(d_1,1–d_1)$. If player 2' s outside option is credible, regardless of player 1's outside option credibility, the resultant equilibrium is given by the partition $(1–d_2,d_2)$.

Proposition [Binmore, Shaked and Sutton, 1988]

Consider a negotiation in which both players can opt out as proposers. Then,

i) If $d_1 \leq \frac{\delta}{1+\delta}$ *and* $d_2 \leq \frac{\delta}{1+\delta}$*, neither outside option is credible and we have a unique equilibrium which coincides again with Rubinstein's equilibrium* $\left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right)$.

ii) If $d_1 > \frac{\delta}{1+\delta}$ *and* $d_2 \leq \delta(1-d_1)$*, the unique SPE is given by the immediate agreement* $\left(1-\delta(1-d_1), \delta(1-d_1)\right)$.

iii) If $d_2 > \delta(1-d_2)$*, the unique SPE is given by the immediate agreement* $(1-d_2, d_2)$[10].

The reason why there isn't a multiplicity of equilibria in this model is that in this case there is not a gap between the best and the worst continuation subgame payoffs. Let us illustrate this point in more detail. Assume player i has an outside option as responder in period t. Her continuation subgame payoff will depend on if her threat of opting out is credible or not in this subgame. If it is credible, she can opt out in period t+2 and, thus, she will get in discounted terms $\delta^2 d_i$. But, given that $d_i > \delta^2 d_i$ for any $0 < \delta < 1$, player i will prefer to opt out at the present than to do it in the future. On the other hand, when player i' s threat of opting out is not credible in the continuation subgame, she will get the Rubinstein's payoff $\frac{\delta}{1+\delta}$. Therefore, player i just need to compare her opting out payoff, d_i, with the continuation subgame payoff that she obtains when her threat of opting out is not credible in this subgame ($\frac{\delta}{1+\delta}$), because when in the continuation subgame opting out is a credible threat, player i will always prefer to opt out in period t ($d_i > \delta^2 d_i$).

[10] Truly, the two conditions that give us the credibility of the outside options are $d_1 \leq \delta(1-d_2)$ or $d_1 > \delta(1-d_2)$ and $d_2 \leq \delta(1-d_1)$ or $d_2 > \delta(1-d_1)$, but solving them mutually we obtain the new conditions of cases i) and ii).

If $d_i > \frac{d}{1+d}$, player i' s optimal strategy is always opt out and if $d_i \leq \frac{d}{1+d}$ player i's optimal strategy is always to accept Rubinstein's offer.

Corollary 3

When in a negotiation neither of the players has the possibility of opting out as proposer, the resultant equilibrium will be unique and efficient.

4) Dependent outside options

Up to now we have assumed fixed outside options, that is, players' opting out payoff is constant and exogenous to the bargaining process. Therefore, in this context when players opted out, they obtained the same value in any period and independent from what the players had done during the bargaining process.

In this section we show that when the assumption of fixed outside options is relaxed, new elements arise in the determinacy of the bargaining outcome. In this sense, given that the literature about this topic is just emerging, our aim is to illustrate through three bargaining models some of these new elements joined to more complex outside options.

Firstly, we study a bargaining situation with an outside option which is a function dependent on time. However, we will assume that this time-dependent function is given exogenously to the model. Secondly, we consider a history-dependent outside option, that is, an outside option which value is determined endogenously by the evolution of the negotiation. Finally, we analyse a bargaining model where players' opting out payoff is uncertain.

4.1) Time-dependent outside options

Rubio (1994) considers a bargaining model where just a player can opt out and only as responder, but incorporating an outside option which is a function dependent on time, that is, by opting out a player obtains different amounts at different times. Therefore, his main purpose is to verify if even non constant between periods outside options alter players' equilibrium strategies.

In particular, suppose player 2 can opt out but only as responder. If she opts out in period t, given that the outside option is dependent on time, the players will get respectively $(0,\delta^t f(t))$, where f(t) is an outside option function without any constraint on its evolution over time[11].

In this context, player 2 might not opt out immediately but in a later period t^* because it is possible that $\delta^t f(t) < \delta^{t^*} f(t^*)$, where $t < t^*$[12]. Player 2's payoff obtained with her opting out threat can be calculated by backward induction from period t^* (given that until that period both players will be bargaining). Thus, the payoff that he gets threatening with opting out in period t^* has two components : (a) her opting out payoff in period t^* $(\delta^{t^*} f(t^*))$ and (b) the payoff that she obtains in the t^*-bargaining periods $(\delta-\delta^2+\delta^3\ldots..+\delta^{t^*})=d*\left(\frac{1+d^{t^*}}{1+d}\right)$.

However, in order to have a credible threat in period t^* player 2 needs that her opting out payoff in period t^* be greater than in any other $t\neq t^*$, that is,

$$t^*\in\left\{\operatorname{argmax}_{t\in\{0,2,4,\ldots\}}(\delta-\delta^2+\delta^3-\ldots..+\delta^t)+f(t)\delta^t\right\}$$

Therefore, the unique SPE of the model is given by the immediate and, thus, efficient agreement,

$$\left(1-\max_{t\in\{0,2,4,\ldots\}\cup(\infty)}\left[(\delta-\delta^2+\ldots..+\delta^t)+f(t)\delta^t\right],\ \max_{t\in\{0,2,4,\ldots\}\cup(\infty)}\left[(\delta-\delta^2+\ldots..+\delta^t)+f(t)\delta^t\right]\right)$$

Notice that the O.O.P is valid in this model because if the outside option is low enough in every period, it will not be credible and, thus, it will not affect players' equilibrium strategies. However, the difference with respect to fixed outside options is that now we just need one period with high enough-valued

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[11] $f(t)\in[0,1]$ and it's assumed that f(t) has a maximum bound. Then, it will be possible to find at least a maximum.

outside option to find the negotiation outcome affected by the outside option. This is so because the existence of that period activates the backward induction process above mentioned, which breaks Rubinstein' s equilibrium strategies.

Proposition [Rubio, 1994]

In an alternating offers bargaining model where just a player can opt out and only as responder, given d Î (0,1) and f(t) Î [0,1] an outside option will affect the bargaining outcome if and only if f(t) > $\frac{d}{1+d}$*, for some t Î {0,2,4,….}.*

On balance, when we assume that the outside options take a different value in every period without introducing any restriction on them (except for f(t)∈(0,1)), the role of the outside options in a negotiation is strengthen because the bargaining outcome can be altered just with one period having a high-enough valued outside option, that is, the O.O.P is strengthen. The main limitation of the present outside option specification is its excessive generality which is related to its lack of economic meaning. Therefore, it might be interesting to introduce some restrictions with economic meaning in the time-dependent outside options. Despite the limitations above described, this class of outside options makes evident that the fixed outside options assumption is too restrictive and that its relaxation could enrich the role of the outside options in a bargaining game. In the rest of this section we examine other two outside option functions, but they will be derived from facts with economic meaning. This new functions will allow us to consider new elements joined to the outside options which have not mentioned up to now despite the fact that they can affect significantly the bargaining outcome.

4.2) "History-dependent" outside options

[12] Notice that with fixed outside options, when player 2 threatened with opting out she preferred to opt out immediately because $\delta^t d < \delta^{t^*} d$ for $\forall t < t^*$.

Compte and Jehiel (1997) considers a bargaining game where players' outside options payoffs depend on what the players do during the negotiation, that is, an outside option outcome is given endogenously by the development of the negotiation. They obtain that when this class of outside options are considered, there are delay agreements. A context where we can find "history-dependent" outside options is a negotiation in presence of an arbitrator.

These authors develop this class of outside options in a concession game, a variant of the partition-offer model analysed through this paper. However, the main results obtained in the concession game are hold in the Rubinstein's alternating offer game. Hence, we can proceed to study the role of endogenous outside options in the context of Rubinstein's model.

Assume the model analysed in the previous section where both players could opt out but only as responders. But now player i's payoff of opting out in period t, instead of being the fixed payoff d_i, is a function $v_i(X,Y)$ that depends on the most generous offers made by the players up to that period.

In particular, to simplify consider the following specification[13] :

$$v_i(X,Y)=Y+\frac{Z}{2}-c$$

where X is the most generous offer that player i has given to player j in earlier periods, that is , X=max (x) between player i's offers (1–x,x). In the same sense, Y is the most generous offer that player j has given to player i in earlier periods, that is, Y=max (y) between player j's offers (y,1–y). On the other hand, Z is the portion of the pie that has not been offered yet, Z=1–X–Y, we assume that this portion is shared equally between both players when any outside option is triggered. Finally, c is a cost associated with the outside option.

Therefore, when any outside option is taken players' payoffs are respectively $\left(v_i(X,Y),v_j(Y,X)\right)=\left(Y+\frac{Z}{2}-c,X+\frac{Z}{2}-c\right)$.

[13] The results of this model are still valid for more general specifications.

Notice that in this model Rubinstein's equilibrium is not in any case an equilibrium. If player i offers the Rubinstein's partition in the first period, player 2 will get $\frac{d}{1+d}$ if she accepts. But player 2 can do better than accepting this payoff :she can reject Rubinstein's partition and opt out. In this case, player 2's payoff would be between $\frac{d}{1+d}$ and 1. If the cost of opting out is not very large, player 2 will prefer this payoff to Rubinstein's.

Therefore, history-outside options provoke that when a player makes an offer, she can increase the outside option of the other player and, thus, the other player have incentives to opt out right away. In order to avoid this incentives, players will begin the negotiation making non-serious offers until they reach an agreement in period t^n where n > 1, that is, there will be delayed agreements.

However, when the costs of opting out (the inefficiency associated with opting out) are sufficiently low, then we can have that the efficiency loss of the delayed agreement is higher than the inefficiency of opting out. Thus, in this situation players will prefer to opt out right away than to reach the delayed agreement.

Formally,

<u>Proposition</u>

Suppose that if players reach an agreement in the negotiation is in period n. Define the inefficiency associated with the delayed agreement at n as $I(n)=1–d^{n–1}$ and the inefficiency of opting out as g=2c. Then,

(i) *If g ³ I(n), in equilibrium players reach an agreement in period n.*

(ii) *If g < I(n) in equilibrium player 2 opts out in the first period.*

Let us illustrate this point with a two-period game. In the first period player 1 is the proposer and player 2 can opt out as responder whereas in period 2 players' roles are reversed. If the second period finishes without any agreement, players get their outside option payoffs (v_1,v_2). Given that in the second period player 1 can opt out, player 2 have to offer her a partition y that makes her indifferent between accepting this offer and opting out :

$$y=v_1(x,y) \rightarrow y = y+\frac{1-x-y}{2} - c \rightarrow y=1-2c-x$$

In equilibrium, player 2's offer will be (y,1–y)=(1–2c–x,2c+x). Then, when player 1 makes an offer (1–x,x) in the first period, any $x > 0$ has two negative effects which weaken her bargaining power. First, it increases her rival's outside payoff given that $v_2(y,x)=x+\frac{1-x}{2}-c$ [14]. Second, it decreases her equilibrium payoff of the next period given that y=1–2c–x. Thus, player 1 will offer to player 2 a payoff of 0 in order to not weaken her bargaining power, that is, (1–x,x)=(1,0).

Taking into account player 1's offer, player 2 will offer in period 2 the partition (y,1–y)=(1–2c,2c). There are two possible equilibria, depending on the parameters values :

(a) If $2c \geq (1-\delta)$, that is, if the inefficiency of opting out is greater than the inefficiency of delaying the agreement one period, player 2 will prefer to wait for the payoff of the second period rather than opt out in the first period because

$$v_2 \leq \delta(1-y) \rightarrow \frac{1}{2}-c \leq d\,2c \rightarrow 2c \geq (1-\delta)$$

In this case, in equilibrium player 2 rejects player 1's offer (1,0), she doesn't opt out in period 1 and offers (1–2c,2c) in period 2, which player 1 accepts.

(b) If $2c < (1-\delta)$, player 2 will prefer to opt out in the first period rather than waiting for the payoff of the next period, given that $v_2 > \delta(1-y)$. Thus, in this case player 2 opts out right away.

[14] We assume that y=0 in player 2's outside option given that in the first period player 2 has not offered any partition yet.

In this section, we have shown that history-dependent outside option in general affect the equilibria of a negotiation, even when there is a cost associated with this outside options. Furthermore, we have seen that history-dependent outside options can justify the existence of delayed agreements. However, this model is only the beginning, further research on this issue will show us new elements that determine the role of the outside options in a negotiation.

4.3) <u>Uncertain outside options</u>

In this section we introduce uncertainty on the sizes of the outside options. Our aim is to show in which way when an element of uncertainty in the outside options is introduced, new elements arise in the determinacy of the equilibria, like the uncertainty revelation period.

In particular, let's consider Ponsatí and Sákovics (1996)'s model where it is assumed that after a responder' s rejection, any player can opt out. But now we suppose that the values of the outside options are random variables $(\tilde{d}_1, \tilde{d}_2)$ distributed according to the cumulative distribution function F_i: $[0,1] \times [0,1] \rightarrow [0,1]$, which is common knowledge between the players.

In the same sense, we assume the existence of three crucial periods in the life of any outside option : (i) T^D, which is the first period in which the outside options are available ; (ii) T^R, which is the period in which the uncertainty is revealed ; (iii) T^M, which is the maturity period of the outside options (In T^{M+1}, they are not available). We will suppose, for simplicity, that $T^D \leq T^R \leq T^M$.

The existence of these three periods is crucial in the determinacy of the equilibria. In order to a better understanding of the bargaining solution, let's divide the game in four parts and proceed going backwards from the fourth stage. We can proceed in this manner because, as we will see just now, the existence of a maturity period breaks the stationarity of the model.

The **fourth stage** ($t > T^M$) begins in T^{M+1} and goes to the end of the negotiation. Because of the fact that in T^{M+1} neither player can opt out, the unique

SPE of the subgame beginning in T^{M+1} is the Rubinstein' s equilibrium : $\left(\frac{1}{1+d},\frac{d}{1+d}\right)$. This equilibrium will be the starting point of the backward induction process that yields the solution of the model.

The **third stage** ($T^R \leq t \leq T^M$) goes from the revelation period to the maturity period. Given that the uncertainty is revealed in T^R, the players know the size of the outside options in this stage. Moreover, they know that they will get Rubinstein' s partition in T^{M+1}. Thus, the players can initiate a backward induction process in the following manner :

(i) If $d_i < \frac{d^2}{1+d}$ and $d_j < \frac{d^2}{1+d}$ [15], players' threats of opting out between T^M and T^R are not credible, because in T^{M+1} they obtain a higher payoff (Rubinstein's partition). Therefore, the unique equilibrium in the subgame beginning in period T^R is again Rubinstein' s equilibrium.

(ii) If any player's outside option is larger than the critical value $\frac{d^2}{1+d}$, the unique equilibrium in T^R is given by either $(1-d_j,d_j)$ or $(d_i,1-d_i)$, depending respectively on if the proposer in period T^R is either player i or player j (assuming that $d_i+d_j \leq 1$). The intuition behind this result is that when a player's opting out payoff is higher than the critical value above mentioned, her opting out threat is credible and, thus, if she is the proposer she takes advantage of this fact giving her rival her reservation value (her rival's outside option payoff), and if she is the responder getting at least her outside option payoff.

(iii) If $d_i+d_j > 1$, then in the unique equilibrium of T^R at least a player opts out. Therefore, the equilibrium partition will be (d_i,d_j).

The players will not know which are their respective outside options payoffs until T^R and, thus, which of the three above described equilibria is realised. However, they have knowledge of the cumulative distribution function of the outside options. Hence, they can compute ex-ante which utility they expect to obtain in period T^R. We will refer to this expected utility as the *strategic value of* *an outside option* (A). In formal terms, assuming that player i is the proposer in period T^R, this concept is given by the vector (A_i,A_j) :

$$A_i=\frac{1}{1+d}\cdot P_1+(1-d_j)\cdot P_2+d_i\cdot P_3 \qquad \text{and} \qquad A_j=\frac{d}{1+d}\cdot P_1+d_j\cdot P_2+d_j\cdot P_3$$

where:

P_1=probability that both players' outside options are lower than the critical value $\frac{d^2}{1+d}$.

P_2=probability that at least one player's outside option is greater than the critical value $\frac{d^2}{1+d}$ and also $d_i+d_j \leq 1$.

P_3=probability that $d_i+d_j > 1$.

Notice that any player's strategic value of an outside option (A_i) is greater than the expected value of the outside option ($E(d_i)$)[16]. Therefore, in the subgame beginning at T^R the unique equilibrium will be given by the players' strategic values vector : (A_i,A_j).

The **second stage** ($T^D \leq t < T^R$) begins in the first period where the outside options become available and ends a period before the revelation period. A possible equilibrium in this stage might consist of solving the game by backward induction, taking as the starting point of this process the unique solution in period T^R (A_i,A_j). Hence, in T^R-1, assuming that player i was the proposer in T^R, player j will offer player i the partition $(\delta A_i, 1-\delta A_i)$. In T^R-2 player i will offer player j the partition $[1-\delta(1-\delta A_i),\delta(1-\delta A_i)]$ and, in general, for $\forall t<T^R$ (where player i is the proponent) this equilibrium will be given by the partition :

[15] If $T^R=T^M$, then the responder's critical value will be $\frac{d}{1+d}$.

[16] This is easily checked by observing that when the outside options take low values, the players get with their strategic values at least Rubinstein's partition payoff.

$$\text{BISVE}=\left(A_i d^{T^R-t} + \frac{1-d^{T^R-t}}{1+d}, 1- A_i d^{T^R-t} - \frac{1-d^{T^R-t}}{1+d} \right)$$

Nevertheless, the partition above mentioned will not always be the equilibrium of this stage. When we are in a period near enough to T^R, players could prefer to wait for the next period payoff rather than to agree the BISVE partition. For instance, assuming that we are in T^R–1 and player j is the proponent in this period. If $A_i + A_j \geq \frac{1}{d}$, the BISVE partition would not constitute an equilibrium because player j will prefer to delay the agreement until T^R, given that $A_i + A_j \geq 1/\delta \rightarrow \delta A_j \geq 1 - \delta A_i$.

On the other hand, let's consider that when $|T^R - t| \rightarrow \infty$, the BISVE partition approaches to the Rubinstein partition $\left(\frac{1}{1+d}, \frac{d}{1+d}\right)$. Therefore, when we move away from T^R, the BISVE partition is closer to Rubinstein's equilibrium. For far enough of T^R periods, a player will opt out when her outside option expected payoff ($E(d_i)$) is greater than the Rubinstein's partition[17]. Hence, again the BISVE partition is not an equilibrium. The new equilibrium consists of either at least a player opting out and, thus, both players obtaining respectively their outside option expected values ($E(d_i)$,$E(d_j)$), or the proposer providing the responder her outside option expected payoff and getting for herself the rest of the pie ($1-E(d_j)$,$E(d_j)$).

To sum up, in the second stage we can find three different equilibria depending on either the parameters of the model and the distance between T^D and T^R: (i) an equilibrium where both players decide to wait for the next period payoff, (ii) the BISVE partition, (iii) an equilibrium where at least a player obtains her outside option expected payoff[18].

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[17] It's not necessary a great distance from the revelation period to find that $E(d_i)$ is higher than the BISVE partition (for instance, in T^R–1 can occur that $E(d_i) > \delta A_i$). We have assumed this great distance in order to clarify the mechanisms arising in the model, given that when we are more distant from the revelation period it is more probable that $E(d_i) >$ BISVE.

[18] Depending on the parameters of the model, it can appear the three equilibria, only two or one. If there exist, the three equilibria proceeding by backward induction it will appear first the equilibrium (i), then the equilibrium (ii) and finally the equilibrium (iii).

The **first stage** ($1 \leq t < T^D$) begins at the beginning of the negotiation and ends a period before the outside options are available. The equilibrium of this stage consists of going backwards from the survival equilibrium of T^D (of the three possible).

In conclusion, the key element that determines the equilibrium in this model is the uncertainty revelation period. Their distance from the beginning of the negotiation will play an important role in the determinacy of the equilibirum.