Lesson 10: Exchange

10.1 The Edgeworth Box

The Edgeworth box is the main tool of analysis to study general equilibrium in an economy with only two consumers (*a* and *b*) and two goods (*x* and *y*). We consider that these two consumers have given endowments of the two goods $\left(\bar{x}, \bar{y}\right)$ and that, because of diversity of tastes, they are willing to exchange these goods between themselves. The purpose of this lesson is to see if such an equilibrium can be characterized and if the price system is an adequate mechanism to reach this equilibrium.

Definitions

Endowments

$\bar{x}_a$: endowment of good *x* held by agent *a*

Similarly, $\bar{y}_a$, $\bar{x}_b$, and $\bar{y}_b$.

$$\bar{x}_a + \bar{x}_b = \bar{x} \quad \text{Total endowment of good } x$$
$$\bar{y}_a + \bar{y}_b = \bar{y} \quad \text{Total endowment of good } y$$

Allocations

x_a: quantity of good x consumed by agent a

Similarly, y_a, x_b, and y_b.

$x_a + x_b = x$ Total quantity consumed of good x
$y_a + y_b = y$ Total quantity consumed of good y

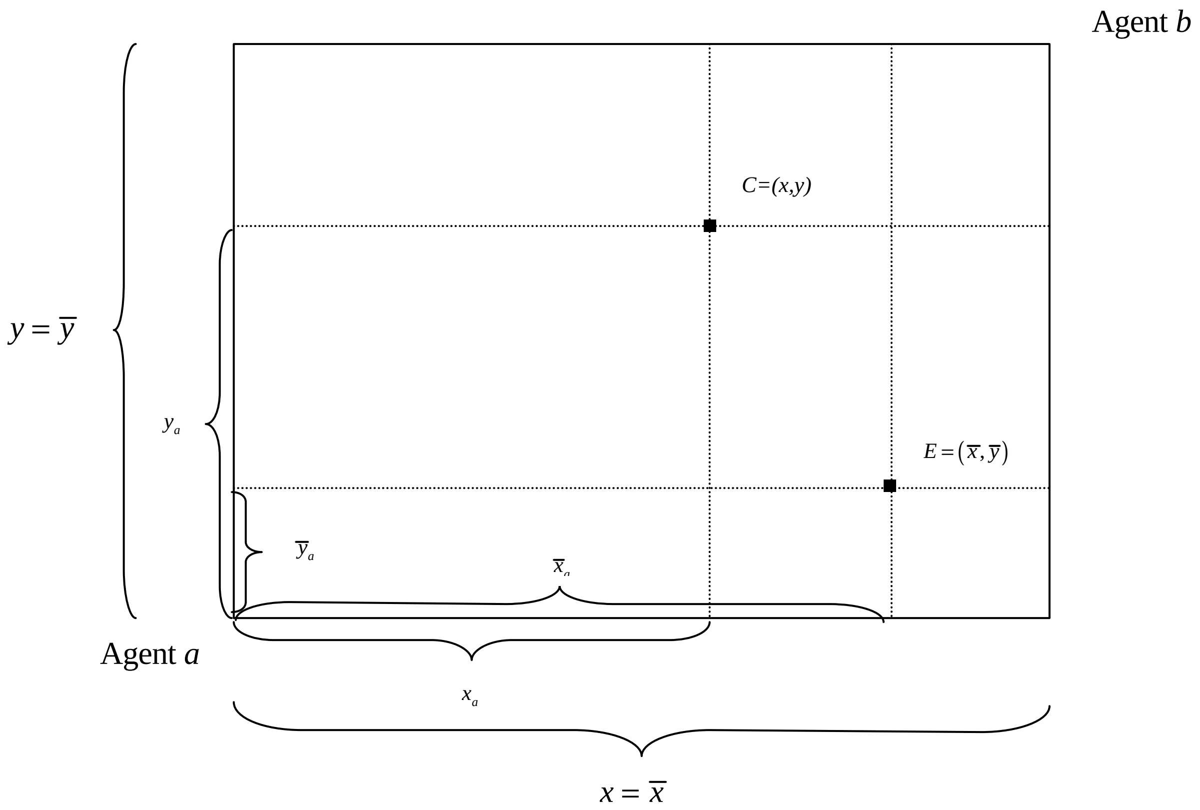
Feasible allocations

$$x \le \overline{x} \quad \text{and} \quad y \le \overline{y}$$
$$x_a + x_b \le \overline{x} \quad \text{and} \quad y_a + y_b \le \overline{y}$$
$$x_a + x_b \le \overline{x}_a + \overline{x}_b \quad \text{and} \quad y_a + y_b \le \overline{y}_a + \overline{y}_b$$

We are going to work in terms of equalities; therefore,

$$x = \overline{x} \quad \text{and} \quad y = \overline{y}$$
$$x_a + x_b = \overline{x} \quad \text{and} \quad y_a + y_b = \overline{y}$$
$$x_a + x_b = \overline{x}_a + \overline{x}_b \quad \text{and} \quad y_a + y_b = \overline{y}_a + \overline{y}_b$$

Graphical representation



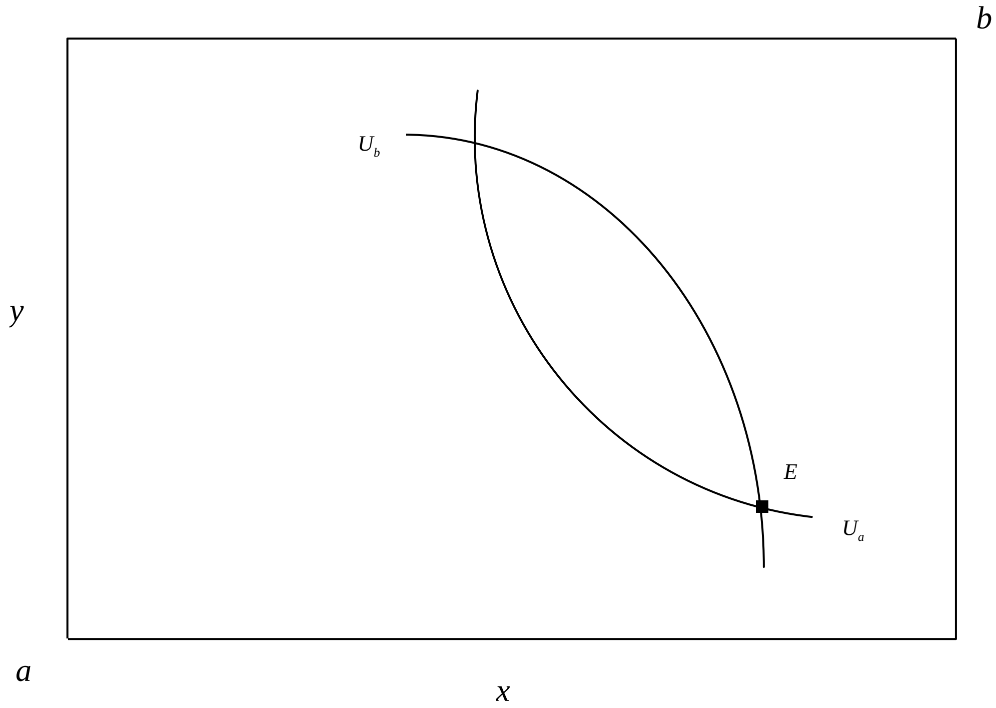
<u>Complete</u> the labels for consumer *b*.

<u>Why is there exchange?</u>

Because the preferences of consumer *a* will normally be different from those of consumer *b*. If tastes differ, most likely both consumers will want to trade goods between each other, because as a result of this trading they will increase their utility. [Illustration: POW camp with French and British prisoners].

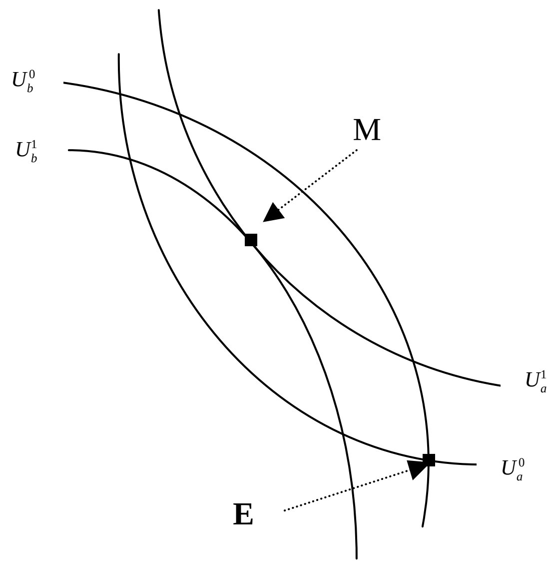
10.2 Pareto efficient allocations

The point is, up to what point will they want to trade? Up to the point where all the utility gains have been exhausted.



If the initial endowment is at point *E*, it is clear that both consumers can increase their utility moving somewhere inside the area limited by the two indifference curves that go through the endowment point.

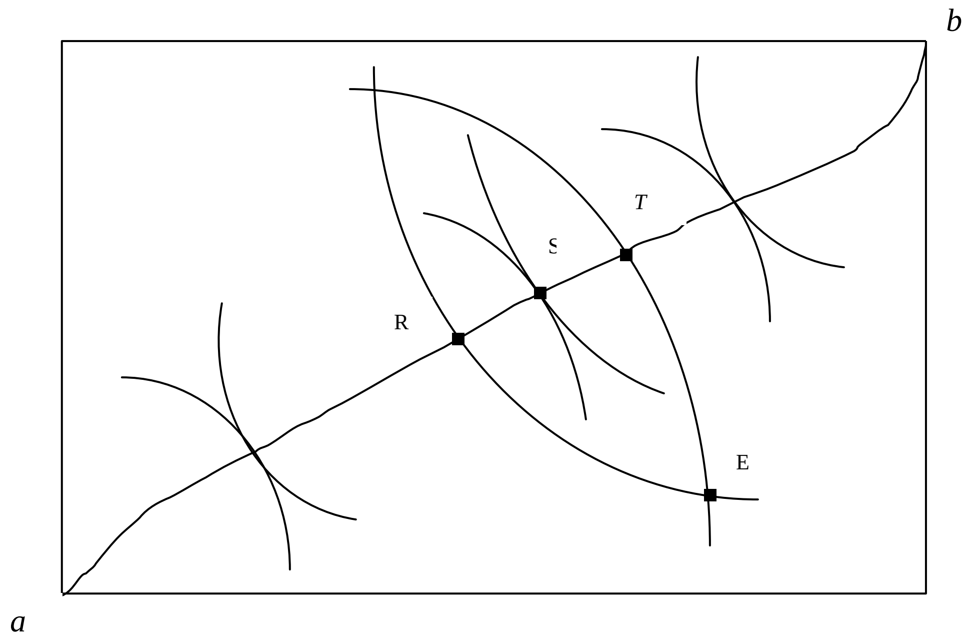
Any point inside this area is an allocation where the utility of both consumers is higher than at E. Also, any point at which the two indifference curves cross each other, is an allocation that has not exhausted all possible gains in utility. In fact, only those points in which the two indifference curves are tangent to each other, are allocations where all gains in utility have been exhausted.



An allocation from which no further gains in utility can be made for <u>both</u> consumers is called a <u>Pareto efficient allocation</u>. In other words, a move from a Pareto efficient allocation, if it generates gains in utility for one consumer, it will necessarily generate loses for the other.

E is not a Pareto efficient allocation. Gains for both consumers can be made. M is a Pareto efficient allocation. From M, any move will involve a lose of utility for at least one consumer.

Within the Edgeworth box there are many Pareto efficient allocations. We call the set of Pareto efficient allocations the contract curve.



In the above diagram, the contract curve is *ab*. If we start at E and we want to end up at an allocation in which all gains have been exhausted, we'll end up on the contract curve. But we can be more precise than this. Under the assumption that consumers do not want to loose utility, we know the that the final position of equilibrium will necessarily be on segment RT. Only in that segment both consumers gain, or one gains and the other does not loose.

Question: Are allocations *a* and *b*, Pareto efficient allocations?

Where, within segment RT, will the final equilibrium be? If we do not make any further consideration, and let the two consumers negotiate, the final equilibrium could be anywhere on this segment (for instance, allocation S could be this equilibrium). One particular equilibrium, which is the one that we are going to discuss here, is the allocation that is obtained as the result of the price system in a competitive market.

10.3 Equilibrium and efficiency

Markets allocate resources through the price system. Suppose that in this economy there is such a market, and also that this market is competitive in the sense that consumers do not control prices. (In an economy with only two agents, this assumption is a bit far fetched, but we will stick to it for expository purposes).

To make things more clear, we will introduce this market through the figure of a hypothetical third agent, which we will call "auctioneer". His mission will be to call prices for goods *x* and *y*, and to check if at this prices an equilibrium is reached. If at the prices called, there is no equilibrium, he will call another set of prices until the equilibrium in question is obtained.

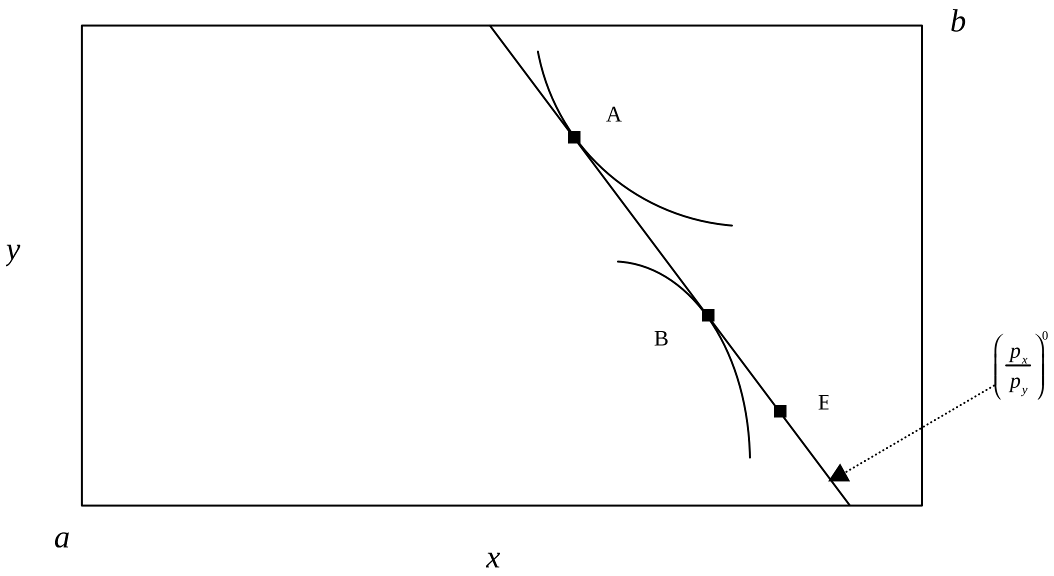
In order to be more precise about this process and about the behaviour of consumers when facing of these prices, we make the following two assumptions, which by now should be familiar to you:

a) We will say that an allocation is an equilibrium allocation, when for all goods total demand equals the total supply.
b) We will suppose that consumers behave so as to maximize their utility subject to their respective budget line.

Suppose the auctioneer calls a given set of prices for goods *x* and *y*. You already know that what matters in economics are relative prices, therefore we will suppose that the auctioneer calls a given relative price $\left(p_x/p_y\right)^0$. This determines the slope of the budget line that will be faced by both consumers. To determine its position, recall what we have discussed in Lesson 6. The budget line will necessarily pass through the endowment point, since both consumers, at whatever prices, will always have available their particular endowment.

Suppose that at this particular relative price, the utility maximizing behaviour leads consumer *a* to allocation A, and consumer *b* to allocation B.

Is this relative price the equilibrium price? Clearly not. At this relative price, the total demand for x is less than the total supply of x (the endowment $\bar{x}$), and the total demand for good y is more than the total supply of y (the endowment $\bar{y}$). At this price, there is excess supply of x and excess demand for y. (Convince yourself in the graph that this is the case).

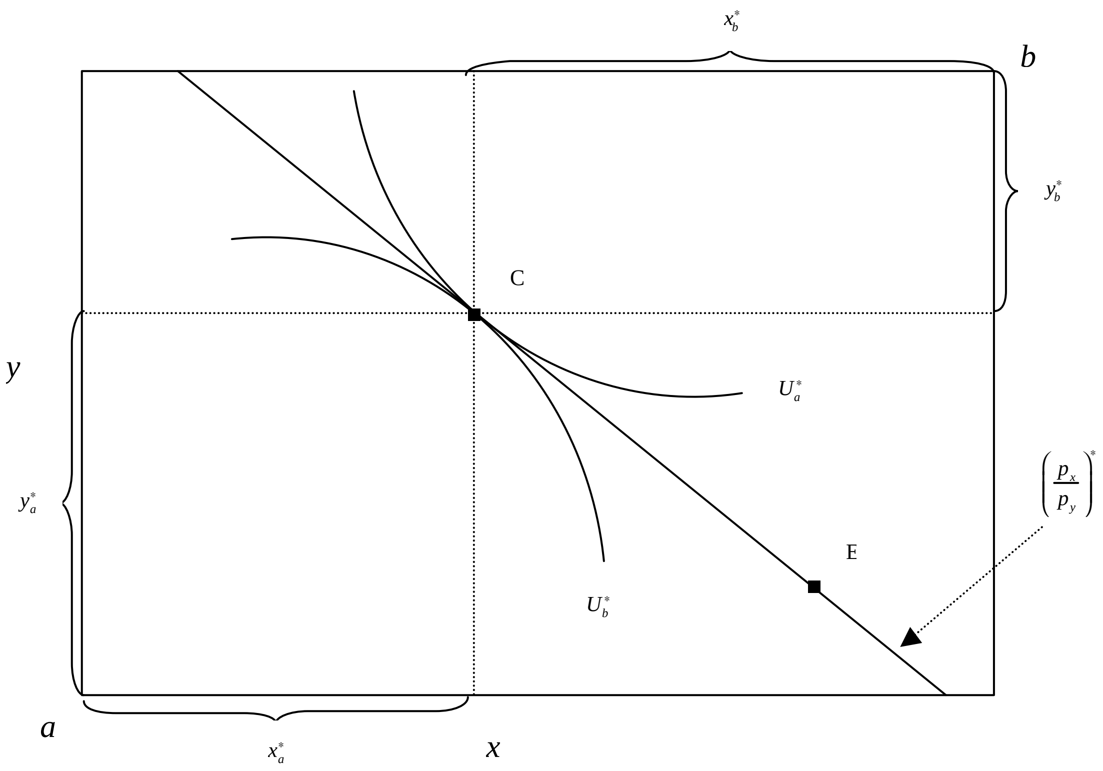


If $\left(p_x/p_y\right)^0$ is not the equilibrium price, what will be the next try of the auctioneer? We will assume, consistently with what we have done in previous lessons and with what we observe in the real world, that the auctioneer will follow this rule:

If there is excess demand for one good the relative price for that good should rise, and if there is excess supply for another good, the relative price of this other good should fall.

This means that the initial relative price for *x* is too high, and therefore that for *y* too low. So, the auctioneer should try a new relative price of *x* (with respect to *y*) lower than the initial one.

If this process of trial and error continues until the point at which for both goods demand equals supply, the auctioneer should arrive at the equilibrium price $\left(p_x / p_y\right)^*$. For this price, the equilibrium allocation is shown in the following graph.



At C, the competitive equilibrium allocation (also called the Walrasian equilibrium), total demand equals total supply for both goods.

$$x_a^* + x_b^* = \bar{x}$$
$$y_a^* + y_b^* = \bar{y}$$

So, we can define the Walrasian equilibrium as that pair of prices $\left(p_x^*, p_y^*\right)$ for which: a) each consumer chooses its most preferred bundle of goods; and b) the choices of all consumers are compatible in the sense that demand equals supply.

We can expand the above two equilibrium conditions, noting that the choice of each consumer for each good is a function of prices (the demand function). (Recall from Lesson 6, that demand functions with fixed endowments only depend on prices, not on income)

$$x_a\left(p_x^*, p_y^*\right) + x_b\left(p_x^*, p_y^*\right) = \bar{x}_a + \bar{x}_b$$
$$y_a\left(p_x^*, p_y^*\right) + y_b\left(p_x^*, p_y^*\right) = \bar{y}_a + \bar{y}_b$$

Or

$$\left[x_a\left(p_x^*, p_y^*\right) - \bar{x}_a\right] + \left[x_b\left(p_x^*, p_y^*\right) - \bar{x}_b\right] = 0$$
$$\left[y_a\left(p_x^*, p_y^*\right) - \bar{y}_a\right] + \left[y_b\left(p_x^*, p_y^*\right) - \bar{y}_b\right] = 0$$

The expressions in brackets are, for each good, excess demand functions of each of the two consumers. Call these excess demand functions *e*(.), then

$$e_a^x\left(p_x^*, p_y^*\right) + e_b^x\left(p_x^*, p_y^*\right) = 0$$
$$e_a^y\left(p_x^*, p_y^*\right) + e_b^y\left(p_x^*, p_y^*\right) = 0$$

Or, if we call the aggregate excess demand functions *z*(.),

$$z_x\left(p_x^*, p_y^*\right) = 0$$
$$z_y\left(p_x^*, p_y^*\right) = 0$$

At the equilibrium prices, the aggregate excess demand functions for both goods must be zero.

In connexion with these functions we can enunciate what is known as the Walras' Law: The value of aggregate excess demand for the whole economy must be zero. (This in fact is valid for any price, not only for the equilibrium price).

$$p_x^* z_x\left(p_x^*, p_y^*\right) + p_y^* z_y\left(p_x^*, p_y^*\right) = 0$$

The proof is very simple and follows directly from the addition of the budget lines of the two consumers (we will not do it here).

But observe that if the above expression holds, and if both prices are positive, then the fact that $z_x\left(p_x^*, p_y^*\right) = 0$ implies that $z_y\left(p_x^*, p_y^*\right)$ will also be zero. If the aggregate excess demand for good *x* is zero, the aggregate excess demand for good y will also be zero. If one market is in equilibrium, the other has to be also in equilibrium. Thus, for *n* goods, we can assure that <u>if *n*-1 markets are in equilibrium, the *n*th market will also be in equilibrium</u>. This is another way to express the Walras' Law.

From all what we have studied in this section we conclude that <u>the competitive equilibrium is Pareto efficient</u>. This is a result that we had already identified in Lesson 9 from partial analysis considerations, and it is good to see that we can also derive it using a general equilibrium approach.

There are many more issues that we leave for Aadvanced Microeconomics in the fourth year: Does a competitive equilibrium exist? Is it unique? Is it stable?