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A new criterion for determining the expansion center for circular-harmonic filters

Pascuala García-Martínez, Javier García, Carlos Ferreira

Departament Interuniversitari d'Òptica, Universitat de València, C/Doctor Moliner 50, E-46100 Burjassot, Spain

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Abstract

A new criterion for locating the expansion center of circular harmonic filters is presented. The innovation consists in the use of the information provided by both the circular harmonic energy map and the peak to correlation energy map of the object to be detected. The choice of an expansion center with a high value of peak to correlation energy ensures a good discrimination capability of the filter. In addition, we choose a point which is a local maximum for the energy map. An improvement of the discrimination ability is obtained with respect to previous methods.

1. Introduction

The matched filter has proved useful for shift invariant pattern recognition [1]. Nevertheless, the ability of this filter to deal with objects which have undergone distortions, such as rotation or scale changes, is very limited [2]. The most widely used method for adding rotation invariance to the matched filter is based on circular harmonic (CH) expansion [3–7]. The circular-harmonic filter is matched to only one circular harmonic component (CHC) of the object. The expansion involves a coordinate transformation of the reference object into polar coordinates. Thus, the functional dependence of the target varies with the location of the polar coordinate system origin, known as the CH expansion center. The choice of an adequate expansion center which will guarantee a good performance of the filter is crucial for a good recognition process.

There are several methods [3,8,9] to select the expansion center. In a first method an arbitrary point is taken as the expansion center to compute the CH filter, then the correlation pattern is calculated, and the maximum intensity peak is determined. The location of the maximum is then used as the new CH expansion center [3]. The process is iterated until the position of the maximum coincides with the expansion center. Unfortunately, this procedure does not always converge to a solution [5]. Furthermore, the solution may not be the optimum one. A second method consists in the construction of a bidimensional energy map of the *m*th order CH component of the object [8]. A maximum on the CH energy map is then chosen as the expansion center. This method presents some problems due to excessive computation time. An algorithm has been recently proposed to increase the speed of the process [9]. It is based on computing the energy of the CHC in the Fourier plane, eliminating the need for expanding the target around each point. Additionally, the process is performed using a simulated annealing algorithm to reduce the number of test points. This method is two orders of magnitude faster than the previous method. Nevertheless, this method is based only on the maximization of the energy of the CHC. In the real case this is not the only important parameter, as it does not take

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into account the discrimination capability of the filter.

We present a new criterion for determining the CH expansion center that consists in using the energy map, and as additional information, a peak to correlation energy (PCE) map. Both maps are calculated simultaneously, almost without requiring an additional computational time. The choice of a local maximum of the energy map as expansion center ensures that the correlation peak occurs at the expansion point. Simultaneously the choice of the expansion center with a high PCE value will optimize the discrimination performance of the filter. Section 2 deals with the expansion center choice. Results are presented in section 3. In section 4 we outline the conclusions.

2. Determination of the expansion center

Defining a polar coordinate system at pont (x, y), the CH expansion of an object function $f(r, \theta)$, expressed in polar coordinates, is

$$f(r, \theta; x, y) = \sum_{m=-\infty}^{\infty} f_m(r; x, y) \exp(im\theta) , \tag{1}$$

where

$$f_m(r; x, y) = \frac{1}{2\pi} \int_0^{2\pi} f(r, \theta; x, y) \exp(-im\theta)\, d\theta . \tag{2}$$

A CH filter matched to a single CHC of the object to be detected, is often used to obtain rotation invariant pattern recognition. If the filter is matched to a CHC of $f(r, \theta)$ and this last function is also taken as the input, in a recognition process, the correlation in the polar coordinate system origin is given by

$$c_m(0, 0; x, y) = 2\pi \int_0^{\infty} |f_m(r; x, y)|^2\, r\, dr . \tag{3}$$

The main advantage of using a circular harmonic reference function is that the intensity correlation output is invariant under a rotation of the input pattern. Thus, rotating the input object by an angle α, the output correlation intensity is rotated by the same angle. The only change is a phase factor $\exp(im\alpha)$ affecting the whole correlation pattern.

The construction of an efficient CH filter of a two-dimensional function $f(r, \theta)$ depends on the expansion center selected. For most of the possible expansion centers, the correlation maximum does not coincide with the expansion center and this may produce sidelobes which are higher than the central correlation values. In addition, the location of the target with different orientations can be problematic.

An expansion center (x_0, y_0) is previously defined to be a proper center for the mth CH order of an object $f(x, y)$, if it satisfies both following conditions [8]:

(a) The central value of the correlation of the target and the CHC obtained with the point (x_0, y_0) is higher than for any other expansion point.

(b) The intensity correlation output with the CHC of the target expanded around the point (x_0, y_0) presents an absolute maximum at (x_0, y_0).

Sheng and Arsenault [8] provide a solution to find the proper center based on the CH energy map. The energy of the mth order CH component of the object, $E_m(x, y)$ is defined as

$$E_m(x, y) = 2\pi \int_0^{\infty} |f_m(r; x, y)|^2\, r\, dr . \tag{4}$$

It should be noted that this value is the same as the central correlation value (see Eq. (3)). Sheng and Arsenault show that the point which maximizes the CH energy map fulfills both the above conditions. Thus, this point is the proper center.

In many cases the choice of the proper center as the expansion center is not the best one. This point could be far from the geometrical center of the object, so the CH filter matched to the CHC of the object, $f(r, \theta)$, would have a poor discrimination capability. To avoid this problem, other points, called subproper centers, may be used as the expansion center. A subproper center is defined as a point which satisfies the second condition of the proper center definition but not the first one. According to this definition, we can find more than one subproper center.

The location of a subproper center is not an easy task either. The only way to determine if a point is a proper center is by testing the correlation plane point by point. To avoid this huge computational time, a local maximum of the CH energy map can be selected as the expansion center. A local maximum of the CH energy map is a local maximum of the correlation plane. If

(x, y) is a local maximum of the CH energy map and (x_1, y_1) is a point near to this local maximum, the correlation value at point (x_1, y_1), using a CH filter with (x, y) as the expansion center, will be

$$|c_{f,m}(x_1, y_1; x, y)|^2$$

$$= (2\pi)^2 \left| \int_0^\infty f_m(r; x_1, y_1)\, f_m^*(r; x, y)\; r\, \mathrm{d}r \right|^2$$

$$\le \left((2\pi) \int_0^\infty |f_m(r; x_1, y_1)|^2\, r\, \mathrm{d}r \right)$$

$$\times \left((2\pi) \int_0^\infty |f_m(r; x, y)|^2\, r\, \mathrm{d}r \right), \qquad (5)$$

where Schwarz inequality has been used. Hence, taking into account Eq. (5) and $E_m(x_1, y_1) \le E_m(x, y)$ we conclude:

$$|C_{f,m}(x_1, y_1; x, y)|^2 \le |C_{f,m}(x, y; x, y)|^2 . \qquad (6)$$

We have taken $E_m(x, y)$ which coincides with the central correlation value. Thus, we have demonstrated that the point (x, y) is a local maximum of the correlation plane. If, in addition, the expansion point provides a high correlation peak value it is likely to be a subproper center (i.e. the central correlation value will be the absolute maximum of the correlation plane).

2.1. Computation of the energy map

To speed up the process we compute the CH energy map in the Fourier plane using the same equations as Prémont and Sheng [9]. This formalism is based on the fact that it is equivalent to calculate the CHC of the object in the object plane or in the Fourier plane. Performing the Fourier transform of a CHC of the object, produces the same amplitude distribution as extracting the CHC of the Fourier transform of the object [10]. A different choice of the expansion center in the object plane, which produces a shift in the object, results in a linear phase factor in the Fourier plane. The main advantage of using the Fourier plane is that the expansion center is always located in the origin. This permits the implementation of more efficient algorithms to compute the energy map.

Let $F(\rho, \phi; 0, 0)$ be the Fourier transform of the non-shifted object, where ρ and ϕ are the polar coordinates in the Fourier plane. The Fourier transform of the object shifted to an arbitrary position (r, θ) in the object plane is

$$F(\rho, \phi; r, \theta) = F(\rho, \phi; 0, 0) \qquad (7)$$

$$\times \exp[-\mathrm{i}\pi\rho r \cos(\theta - \phi)] . \qquad (7)$$

The CH expansion of Eq. (7) then yields

$$F_m(\rho, \phi; r, \theta) = F_m(\rho; r, \theta)\, \exp(\mathrm{i}m\phi) , \qquad (8)$$

with

$$F_m(\rho; r, \theta) = \sum_{n=-\infty}^{\infty} F_n(\rho; 0, 0)\, J_{m-n}(2\pi r\rho)$$

$$\times \exp[-\mathrm{i}(m-n)\ (\theta + \pi/2)] . \qquad (9)$$

The correspondent filter is the conjugated of the CH component in the Fourier plane. The mth order radial function, $F_m(\rho; r, \phi)$ of the Fourier transform of the object shifted to the point (r, θ) is expressed as an infinite linear combination of the CH functions $F_n(\rho; 0, 0)$ of the nonshifted object Fourier transform. This makes calculation of the energy map easier, because the CH expansion in each point (r, θ) of the object is not required, as it is for calculation in the object plane.

The mth CH energy map is then obtained as

$$E_m(r, \theta) = 2\pi \int_0^\infty |F_m(\rho; r, \theta)|^2 \rho\, \mathrm{d}\rho . \qquad (10)$$

In general, the energy map calculation can be useful for proper center choice, but it does not provide a definitive method. We go on to complement the information provided by the CH energy map with the peak to correlation energy (PCE) map.

2.2. Peak to correlation energy map

The PCE is a quality parameter [11] which is defined as the central correlation intensity value over total intensity at the correlation plane. It measures the intensity concentration in the correlation plane. In the case under consideration of a mth order of a CH filter, the PCE can be written as

$$\mathrm{PCE}=\frac{|C_m(0,0;x,y)|^2}{\int\int_\Sigma |C_m(x',y';x,y)|^2\,\mathrm{d}x'\,\mathrm{d}y'}=\frac{E_m(x,y)}{\int\int_\Sigma |C_m(x',y';x,y)|^2\,\mathrm{d}x'\,\mathrm{d}y'}\,. \tag{11}$$

A high value of PCE is connected with a narrow maximum in the correlation plane. To evaluate the denominator we perform the correlation between $F(\rho, \phi)$ as input image, given by Eq. (8), and a filter obtained from Eq. (9). The energy of the correlation plane ε_i, for each point (x, y), is

$$\varepsilon_i=\int_0^{\infty}\int_0^{2\pi}|F_m^*(\rho)\times\exp(-im\phi)\,F(\rho,\phi)|^2\,\rho\,\mathrm{d}\rho\,\mathrm{d}\phi=\int_0^{\infty}F_m^*(\rho)\left(\int_0^{2\pi}|F(\rho,\phi)|^2\,\mathrm{d}\phi\right)F_m(\rho)\,\rho\,\mathrm{d}\rho\,, \tag{12}$$

where the radial symmetry, in intensity, of the CH expansion has been taken into account.

The CH energy map and the PCE map are computed together in the same algorithm. We choose the maximum of the PCE map as the expansion center to perform the CH filter. This election ensures a high discrimination ability of the CH filter in the correlation process but it is not the only step to be performed. The election of the expansion center using the PCE map should be complemented with the information provided by the CH energy map in order to optimize the intensity correlation output as well. In fact, if the maximum of the PCE map chosen is a local maximum of the CH energy map, probably, this point will be a subproper center and then, the center correlation peak will have a high value. Otherwise, if the PCE maximum does not coincide with a local maximum of the energy map, we choose the location of the local maximum of the energy map with the highest PCE value as the expansion center. Using this method a CH filter with a high discrimination ability will be obtained, so decreasing the possibility of false alarms with respect to the other method where the expansion center is chosen based on the maximum of the CH energy map.

3. Results

In this section, computer simulations are presented in order to demonstrate the performance of the proposed criterion to determine the expansion center. Images of 128×128 pixels size are used both for the input scene and for the target to be detected.

First, we consider the scene shown in Fig. 1. In this image there are objects of two different shapes with three instances of the target (marked with an arrow) at different orientations.

The key point in the rotation invariant recognition procedure is to analyze the target pattern to find the expansion order and the expansion center for the circular harmonic filter. The selection of a good expansion order is problematic. Only a complete analysis of the energy and the PCE maps or the comparison among different orders, can facilitate the right choice. To avoid this tedious procedure we can choose an arbitrary expansion order. Firstly, we choose $m=1$ as an arbitrary expansion order and compare the results obtained applying the new criterion proposed in this paper with the energy map based one. Figs. 2a and 2b show the correlation output obtained when using the CH filters computed by both methods respectively. These figures show that the use of the new criterion to determine the expansion center provides a better discrimination than when the maximum of the CH energy has been used. The correlation obtained in this last case (Fig. 2a) presents high sidelobe values which may make recognition of the target difficult. We obtain a detection

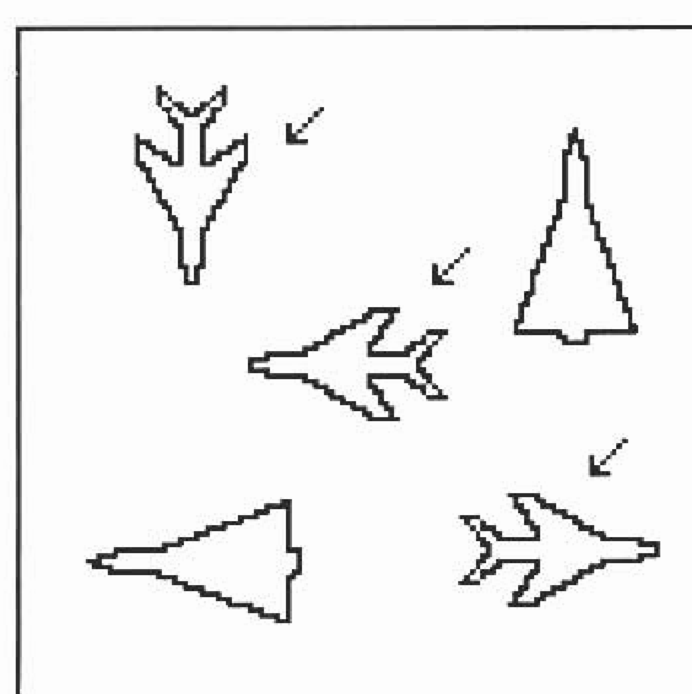

Fig. 1. Input scene containing two types of outlined airplanes. The target is indicated by an arrow.

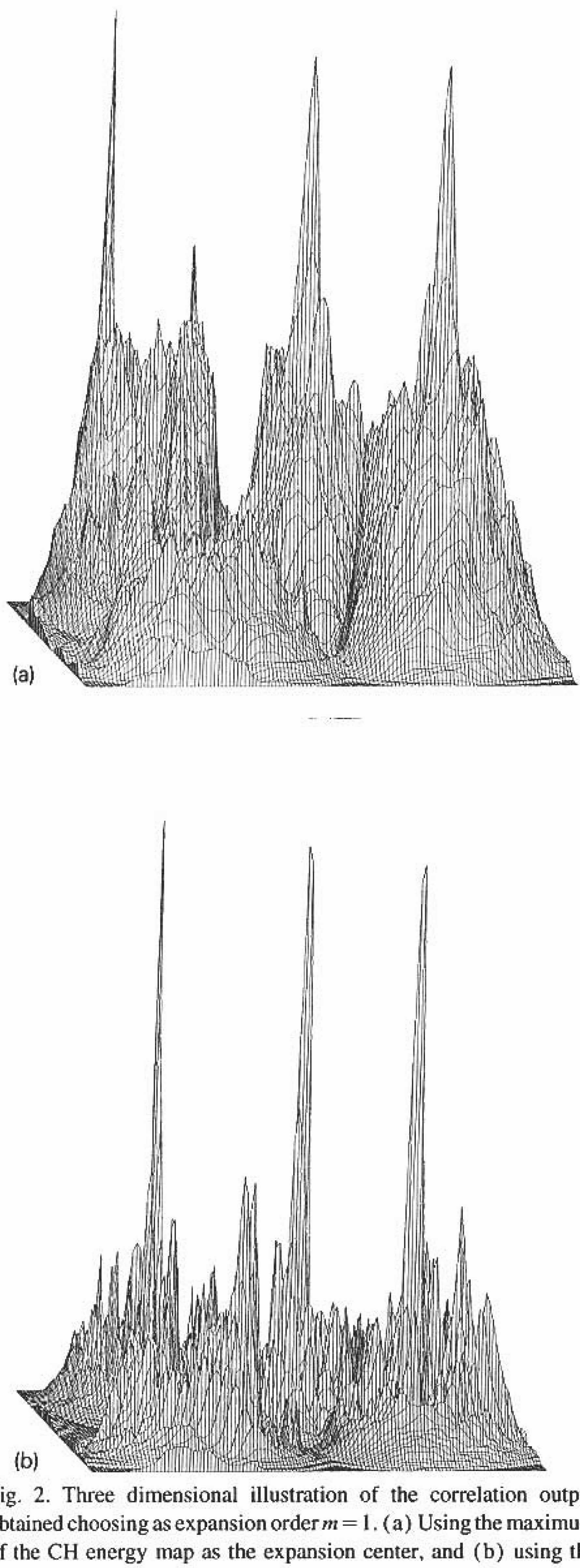

Fig. 2. Three dimensional illustration of the correlation output obtained choosing as expansion order $m = 1$. (a) Using the maximum of the CH energy map as the expansion center, and (b) using the new criterion to determine the expansion center.

threshold of 47% when the maximum of the PCE map is used as the expansion center, as opposed to a threshold of 62% with the maximum of the CH energy map.

Secondly, the best expansion order is sought, computing the CH energy map and the PCE map for orders ranging from 0 to 10. We choose an expansion order which presents a sharp maximum of the energy map located relatively close to the center of the object. For the target of Fig. 1, order $m = 3$ is a good choice. In the CH energy map the maximum can be observed at point (64,73). This point is the proper center for this order and can be a good choice as expansion center. In the PCE map, the maximum is placed at point (64,63). Two CH filters have been computed, the expansion center being in one of them that provided by the energy map, and in the other that provided by the PCE map. In this case similar output correlations are obtained, although they are slightly better when the expansion center has been chosen based on the PCE map. With the PCE map based CH filter, a threshold of 28% is necessary to discriminate the target from the other objects in the scene of Fig. 1. This threshold slightly increases up to 30% when the energy map based CH filter is used. Nevertheless, this second experiment does not permit any conclusions to be reached about the discrimination ability when using the new criterion, because the true target and the other objects in the scene are very different in shape. For this reason, we take another scene with an object very similar to the target that we want to detect, in order to greatly increase the probability of a false alarm in the correlation output.

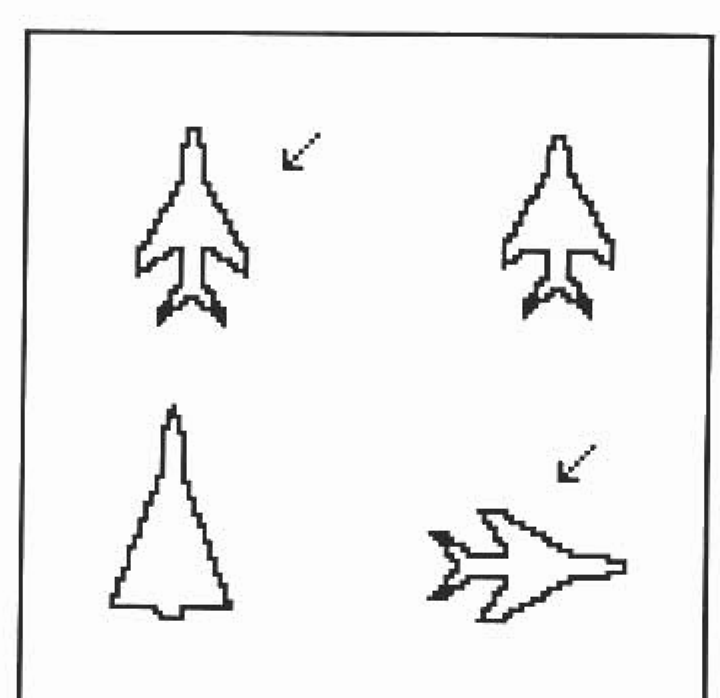

Fig. 3. Input scene containing three types of outlined airplanes. An object very similar to the target is included.

Such a scene is shown in Fig. 3. The target, marked with an arrow, appears at two different orientations. The object similar to the target is placed at the upper right corner of the scene. Using the CH filters previously computed, with expansion order $m=3$, and, as expansion centers, the points provided by the energy map and the PCE map, we obtained the output correlations shown in Figs. 4a and 4b, respectively. As can be observed, the behaviour of the PCE map based filter is better than that of the CH energy map filter. In the last case one false alarm appears while in the first one a threshold of 58% is enough to reject the false alarm.

4. Conclusions

Determining the CH expansion center is an important task in rotation invariant pattern recognition with CH filters. In this article, a new procedure to choose the CH expansion center is proposed. It is based on the use of both the PCE map and the CH energy map. The expansion center is selected as the local maximum of the CH energy map with the highest PCE value. The calculations are made in the Fourier plane, where the CH energy map can be calculated faster. The PCE map is simultaneously calculated with almost no additional computational load.

The results obtained when using this method show an increase of the discrimination capabilities of the CH filter. It provides fair results even in the case of expansion orders which are not appropriate for the target. When using a suitable order the performance of the filter is enhanced especially for the case of similar objects.

Further investigations along the use of different quality parameters can be introduced to improve the performance of the CH filters and the rotation invariance process for optical pattern recognition.

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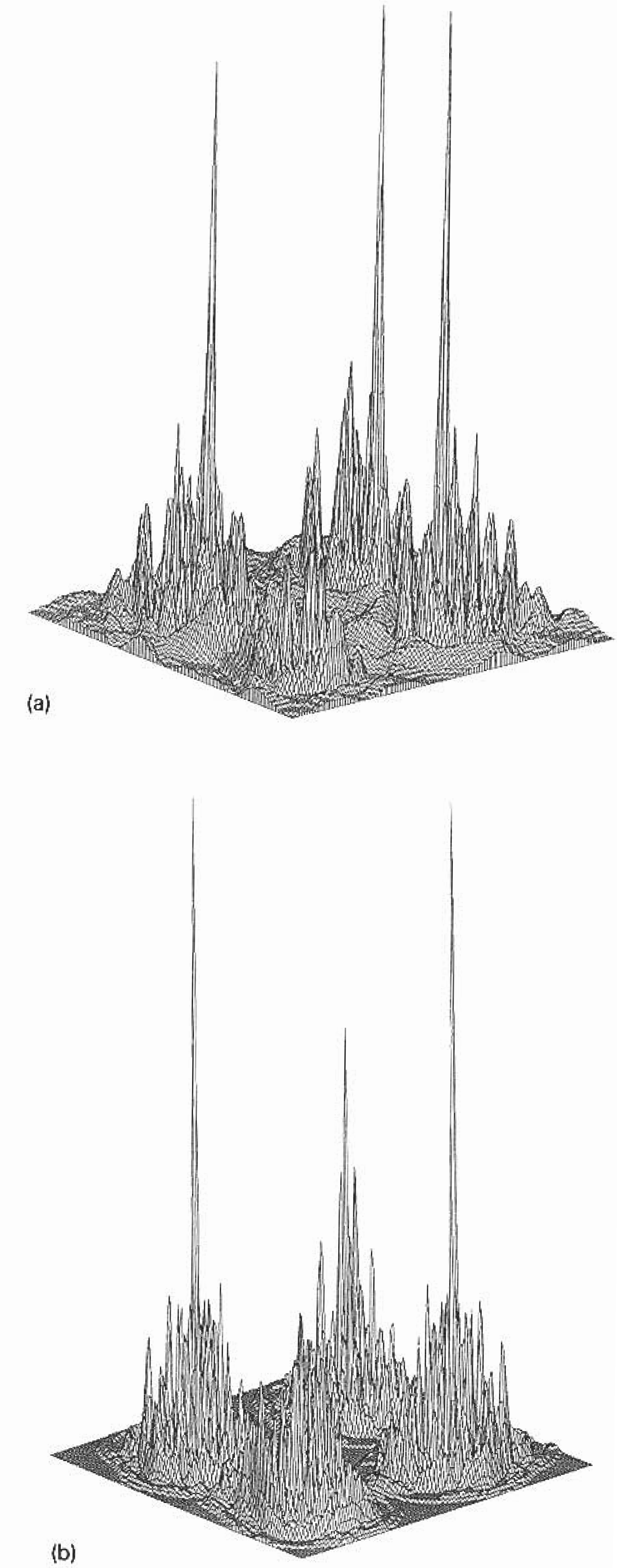

Fig. 4. Three dimensional illustration of the correlation outpu obtained choosing as expansion order $m=3$, (a) using the maximur of the CH energy map as the expansion center, and (b) using th new criterion to determine the expansion center.

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