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Twist angle determination in liquid crystal displays by location of local adiabatic points

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Abstract

In this work we present a method for the determination of the twist angle of an arbitrary twisted nematic liquid crystal spatial light modulator. The method is based on the location of local adiabatic points, i.e., situations in which the liquid crystal SLM acts only as a rotation device. For these cases, the rotation induced on the polarization of the incident beam is equal to the twist angle. Consequently, the twist angle can be determined with high precision. We show that local adiabatic regime may be achieved in two ways, either by changing the incident beam wavelength, or by applying a voltage to the electrodes of the display. However, the simple model that describes the SLM in the off-state, may break down when a voltage is applied to the display, and it may affect the local adiabatic behaviour. We present theoretical and experimental results. © 1998 Elsevier Science B.V. All rights reserved.

Keywords: Spatial light modulators; Twisted nematic liquid crystals; Twist angle

1. Introduction

Twisted nematic liquid crystal spatial light modulators (TNLC-SLMs) are one kind of electro-optics display widely used for optical processors [1] and optical correlators [2]. In general, these devices produce coupled amplitude and phase modulations versus applied voltage [3,4]. However, under proper conditions of polarization and proper ranges of voltage, amplitude-mostly or phase-mostly configurations can be obtained. In this sense, the transmission of eigenpolarization states has been studied to obtain phase-only modulation [5–7]. A prior knowledge of the physical parameters of the TNLC-SLM is necessary to obtain the Jones eigenvector that produces the phase-mostly modulation.

Lu and Saleh [8] developed a model to describe the liquid crystal display (LCD) based on the Jones matrix theory. They obtained a Jones matrix that is referred to the coordinate system where the LC director at the input surface of the SLM is oriented parallel to the coordinate axis. However, in general, the location of the LC director relative to the laboratory axes (ψ_D) is unknown for the user and it must be determined. In addition, two other parameters control the modulation: the twist angle (α) and the birefringence of the liquid crystal (β).

Several methods have been proposed in the literature for the determination of these parameters. Soutar and Lu [9] proposed a technique based on a measurement of the intensity transmitted through the LCD inserted between two polarizers, when both polarizers are rotated simultaneously either parallel or perpendicular. The values of α, β and ψ_D are obtained by a curve fitting procedure. However, this technique may produce ambiguities in the results. The use of different wavelengths has been proposed [10] to overcome these ambiguities. Yamauchi and Eiju [11] determined experimentally the LCD Jones matrix from intensity transmittance measurements and obtained the LCD parameters using numerical fits of these curves.

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Then they were able to predict the polarizer and analyzer angles that optimize the phase modulation. Recently, other procedures to measure α and β have been proposed based on Stokes parameter measurements [12], and based on a channeled spectrum principle [13].

In this work we present a simple method to measure independently the twist angle α, by achieving local adiabatic regime of the LCD. An independent measurement of α simplifies the previous methods because one of the parameters is already known. The local adiabatic regime occurs when the magnitude γ, defined as $\gamma=\sqrt{\alpha^2+\beta^2}$, is equal to $n\pi$, where n is an integer number. Then the rotation in the polarization induced by the LCD is a direct measurement of the twist angle. Goncalves-Neto et al. [14] reported this situation for the measurement of the parameters of the LCD and show that they could achieve it by applying voltage to the LCD. However, as they say, when a high voltage is applied to the LCD, the assumptions made by Lu and Saleh in their model (twist angle linear with respect to depth of LCD, and effective birefringence constant for every layer of LCD) may fail [14,15]. Here, we analyze the local adiabatic regime in terms of Jones matrix formalism and show how it can be achieved either by changing the voltage, or by changing the wavelength of the incident beam. This latter case is more appropriate because no voltage is applied to the LCD and the assumptions of the model by Lu and Saleh are correct.

In Section 2, we review the Jones matrix model the LCD [8]. In Section 3, we examine in detail this matrix model for various situations of the LCD, in particular the adiabatic regime and the local adiabatic points. In Section 4, we give experimental results that demonstrate that local adiabatic behavior is obtained by changing the incident wavelength, and also by applying voltage to the LCD. However, we show that when a voltage is applied to the LCD, the model by Lu and Saleh [8] may break down and it affects the local adiabatic behaviour. Finally, in Section 5 we present an additional experiment to determine the sense of the twist angle.

2. Jones matrix theory

Yariv and Yeh [16] developed the Jones matrix for a twisted anisotropic media by regarding it as a stack of birefringent layers, each one slightly twisted with respect to the previous one. Each layer is considered as a uniaxial birefringent plate. They assumed that the fast and slow axes of the first layer coincide with the x and y axes of the coordinate system. The result is the following Jones matrix

$$\mathbf{M}_{\mathrm{LCD}}(\alpha,\beta)=\exp[-i(\phi+\beta)]\mathbf{R}(-\alpha)\mathbf{M}(\alpha,\beta), \quad (1)$$

where $\mathbf{R}(\theta)$ is the 2×2 rotation matrix given by

$$\mathbf{R}(\theta)=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}, \quad (2)$$

and $\mathbf{M}(\alpha,\beta)$ is given by

$$\mathbf{M}(\alpha,\beta)=\begin{bmatrix}X-iY & Z\\ -Z & X+iY\end{bmatrix}, \quad (3)$$

Here $X=\cos(\gamma)$, $Y=\beta\sin(\gamma)/\gamma$, $Z=\alpha\sin(\gamma)/\gamma$ and $\gamma^2=\alpha^2+\beta^2$. The birefringence β is defined as $\beta=\pi d\Delta n/\lambda$ where d is the thickness of the display, λ is the incident wavelength, and Δn is the difference between the ordinary (n_o) and extraordinary (n_e) indices of refraction for the liquid crystal molecules. The constant phase ϕ is defined as $\phi=2\pi dn_o/\lambda$. Because ϕ is a constant phase which is not affected when a voltage is applied to the display, it will be neglected in the following.

Lu and Saleh [8] extended this model to the LCD. They assumed that when a voltage is applied to the electrodes of the display, the LC molecules tilt towards the direction of the propagation (z-axis). Then, the effective birefringence changes as a function of voltage. They define a new parameter $\beta(V)$ given by $\beta(V)=\pi d\Delta n(V)/\lambda$ where now $\Delta n(V)$ is a voltage-dependent magnitude. Consequently, the same matrix $\mathbf{M}_{\mathrm{LCD}}$ describes the LCD simply by replacing β by $\beta(V)$. This is a parameter that decreases as the voltage increases. Let β_{max} denote the value $\beta(V)$ for the off-state ($V=0$).

In general, the LC director at the input surface is not oriented parallel to the laboratory frame and the matrix $\mathbf{M}_{\mathrm{LCD}}$ must be transformed by a rotation. Consequently, three parameters are of interest to determine the Jones matrix of the LCD: the twist angle (α), the maximum birefringence (β_{max}) and the orientation of the director at the input surface of the display (ψ_D).

3. Adiabatic and local adiabatic regimes

Initial works [17,18] on the determination of the physical parameters of the LCD were based on the assumption of the adiabatic following approximation. This is a valid approximation for thick LCDs. In this situation, the value of β is much greater than α, and the matrix $\mathbf{M}_{\mathrm{LCD}}$ may be approximated by

$$\mathbf{M}_{\mathrm{LCD}}(\alpha,\beta\gg\alpha)=\exp(-i\beta)\mathbf{R}(-\alpha)\begin{bmatrix}\exp(-i\beta) & 0\\ 0 & \exp(+i\beta)\end{bmatrix}. \quad (4)$$

Then, according to Yariv and Yeh [16], the action of the LCD can be divided in two parts. First, a phase retardation matrix of phase shift 2β operates on the Jones vector of the incident wave. Second, the operation of the rotation matrix is to rotate the Jones vector by an angle α. In this situation, the determination of the orientation of the LC director becomes easy. Incident linearly polarized light in the direction parallel or perpendicular to the LC director will emerge again linearly polarized, but rotated by an angle α. The LC director orientation can be obtained by searching for the maximum extinction when the LCD is placed between two polarizers, and the twist angle is equal to the rotation induced on the direction of polarization.

However, for thin LCDs the adiabatic following approximation is not valid. Nevertheless, there are certain values of β for which the matrix $\mathbf{M}_{\mathrm{LCD}}$ has an easy expression, those for which $\gamma = n\pi$, where n is an integer. Then the matrix $\mathbf{M}(\alpha,\beta)$ reduces to an identity matrix and the LCD matrix is

$$\mathbf{M}_{\mathrm{LCD}}(\alpha,\gamma=n\pi) = \exp(-i\beta)\mathbf{R}(-\alpha)\begin{bmatrix}(-1)^n & 0\\ 0 & (-1)^n\end{bmatrix}. \tag{5}$$

In this situation, the action of LCD reduces to a rotation of angle α on the polarization of the incident beam, without changing the state of polarization. We refer to these situations as *local adiabatic points*. The difference between this situation and the previous one is that, in a local adiabatic point (Eq. (5)), linearly polarized incident light will remain linearly polarized but rotated, independently of the orientation. In the adiabatic regime for thick LCDs (Eq. (4)), emerging light will remain linearly polarized only when the orientation of the polarization is either parallel or perpendicular to the LCD director at the input surface.

Another special case occurs for the maximum voltage. In this case, β tends to zero and the matrix $\mathbf{M}(\alpha,\beta)$ becomes a rotation matrix of angle α. Consequently, the Jones matrix $\mathbf{M}_{\mathrm{LCD}}$ tends to be

$$\mathbf{M}_{\mathrm{LCD}}(\alpha,\beta=0) = \mathbf{R}(-\alpha)\mathbf{R}(+\alpha) = \mathbf{I}, \tag{6}$$

where $\mathbf{I}$ represents the identity matrix. As expected for this situation, the output beam polarization is identical to that of the input incident beam.

In order to obtain the local adiabatic regime, it is necessary to control the value of β. It can be changed in two ways: by changing the wavelength of the incident beam, or by applying voltage to the LCD. We inspect these two ways to look for local adiabatic points.

4. Location of local adiabatic points

Here we analyze the polarization ellipse emerging from the LCD as a function of α, β and the orientation of the input polarizer. Let us assume that the LCD is inserted between two linear polarizers, with angles φ_1 and φ_2 with respect to the LCD director at the input surface of the LCD. The normalized transmission of the system is given by [8]

$$T = \left[X\cos(\varphi_1 - \varphi_2 + \alpha) + Z\sin(\varphi_1 - \varphi_2 + \alpha)\right]^2 + \left[Y\cos(\varphi_1 + \varphi_2 - \alpha)\right]^2. \tag{7}$$

In order to characterize the polarization ellipse emerging from the LCD, we performed the following experiment. For a given value of φ_1 we search for the values of φ_2 that give maximum and minimum transmission. These angles correspond to the major and minor axes of the polarization ellipse. Fig. 1 shows the result of this simulation for a twisted nematic LCD with a twist angle $\alpha = 75°$. Three different curves are presented for three different values of β which correspond to $\gamma = 2\pi$, $\gamma = 2.1\pi$ and $\gamma = 2.2\pi$. Fig. 1(a) shows the maximum and minimum transmission vs. the input angle for these three points of the LCD. When the LCD is on the local adiabatic point $\gamma = 2\pi$, the emerging light remains always linearly polarized, independently of the angle φ_1. However, when the

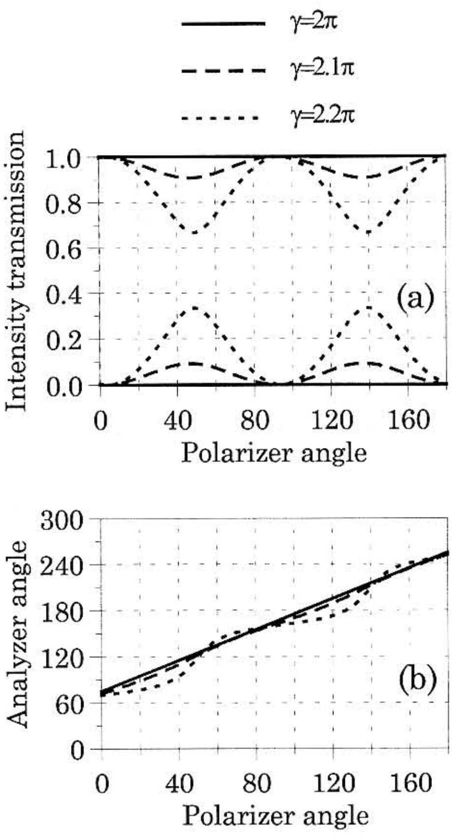

Fig. 1. Theoretical results for a LCD with twist angle $\alpha = 75°$, for values $\gamma = 2\pi$, $\gamma = 2.1\pi$, and $\gamma = 2.2\pi$. (a) Maximum and minimum transmission versus φ_1. (b) Angle φ_2 that gives maximum transmission versus φ_1.

LCD is on an arbitrary point, great variations are observed as a function of φ_1. Fig. 1(b) shows the angle φ_2 that gives the maximum transmission as a function of φ_1, i.e., the angle of the major axis of the ellipse. When the LCD is on a local adiabatic point, φ_2 is a linear function of φ_1, and the difference between them is always α. In particular, for this simulation, $\varphi_2 = 75°$ is obtained for $\varphi_1 = 0$. However, when the LCD is not on an adiabatic point, then the relation of φ_2 with respect to φ_1 is not linear.

Consequently, the twist angle may be obtained with high accuracy by measuring the rotation of light when the LCD is on a local adiabatic point. It is important to note that even when the LCD is not on an adiabatic point, there are always two orientations of the incident polarization for which the light remains linearly polarized at the exit [13]. These two orientations, φ_1^{LP}, are the solution of the equation $\varphi_1^{LP} = (1/2)\arctan[Z/X]$. Nevertheless, although light remains linearly polarized, the rotation in the polarization plane is equal to $\pi/2 + \alpha - 2\varphi_1^{LP}$, which is a magnitude that depends on φ_1^{LP}. Consequently, this rotation is not useful for the measurement of α. Only when the LCD is on a local adiabatic point will the rotation in the polarization plane be equal to α.

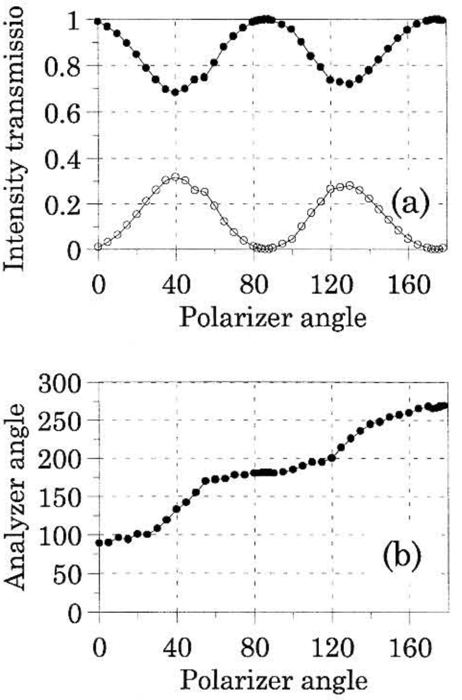

Fig. 2. Experimental results obtained with the LCD in the off-state and $\lambda = 514$ nm. (a) Maximum and minimum transmission versus input polarizer angle. (b) Analyzer angle that gives maximum transmission versus input polarizer angle.

4.1. Adiabatic points obtained by change of the incident wavelength

We use this proposed technique to measure the twist angle of a twisted nematic LCD extracted from a video-projector, model Epson VP-100PS. This LCD corresponds to blue illumination inside the video-projector. In order to perform the experiment, the LCD is inserted between a polarizer–analyzer pair. The system is illuminated with circularly polarized collimated beam in order to maintain a constant incident intensity when rotating the first polarizer. Then the following experiment is performed: (1) input polarizer is placed at a given angle, (2) analyzer is rotated in an attempt to achieve the minimum transmission for each orientation, (3) maximum transmission is also measured by rotating the analyzer 90°. Steps 2 and 3 are repeated for each position of the input polarizer.

Firstly, we analyzed the LCD in the off-state. In this situation, the value of β_{max} determines whether the LCD is on a local adiabatic point. We can change the value of β_{max} by changing the wavelength of the light. The shorter the wavelength, the greater is the value of β_{max}. We performed experiments with green illumination from an Ar laser ($\lambda = 514$ nm), and with red illumination from a He–Ne laser ($\lambda = 633$ nm).

Fig. 2 shows the experimental results obtained for $\lambda = 514$ nm. Fig. 2(a) shows the evolution of the maximum and minimum transmission versus input polarizer angle, while Fig. 2(b) shows the evolution of the analyzer angle that gives the maximum transmission, i.e., the orientation of the major axis of the ellipse. Great variations are observed in the maximum and minimum transmission. In addition, the analyzer angle is not linear with respect to polarizer angle. From these results, we can conclude that the LCD is not acting in an adiabatic point for this wavelength.

Fig. 3 shows the results for $\lambda = 633$ nm. For this wavelength almost no variation of maximum and minimum transmission vs. polarizer angle is observed. In addition, a linear relation is obtained for the analyzer versus polarizer angle. These results show that for this wavelength, the LCD in the off-state is on a local adiabatic point. By measuring the rotation of the polarization in this situation, the possible values for the twist angle are $\alpha = 103 \pm 1$ or $\alpha = -77 \pm 1°$. These two possible values come from the ambiguity in the sense of the rotation.

These results show that local adiabatic regime can be obtained in the off-state by changing the wavelength. This is in accordance with the model by Lu and Saleh [8]. For our LCD, it has been casual that He–Ne laser wavelength gives the adiabatic point. In general, a continuous change in wavelength can be necessary if none of the available wavelengths from lasers give the local adiabatic behavior. Light from a monochromator could be used in the experiments.

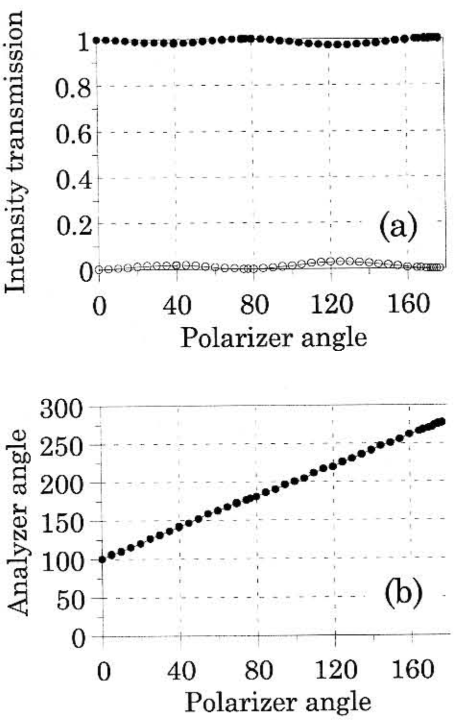

Fig. 3. Experimental results obtained with the LCD in the off-state and $\lambda = 633$ nm. (a) Maximum and minimum transmission versus input polarizer angle. (b) Analyzer angle that gives maximum transmission versus input polarizer angle.

4.2. Experimental verification

In order to verify the value of the twist angle, we measured the transmission of the LCD versus voltage. To correctly match experimental curves to theoretical predictions, it is necessary to know the orientation of the LC director relative to the laboratory frame (ψ_D). We used the Soutar and Lu method [9] to determine it. This technique is based on a curve fitting procedure of transmission curves obtained when both polarizers are rotated simultaneously either parallel or perpendicular. From these two experimental curves it is possible to determine the values for ψ_D, β and α. This is an example of a technique that is notably simplified by a prior knowledge of α. For our LCD, we obtained that the LC director at the input surface is oriented at $\psi_D = 80.5°$ with respect to the vertical laboratory frame.

We selected a configuration of the polarizers angles $\varphi_1 = 0$ and $\varphi_2 = \alpha$. In this configuration, input polarization is selected parallel to the LC director at the input surface, and output analyzer selects the component of the emerging light parallel to the LC director at the output surface. Because the signal is controlled with a video card, the voltage is dependent on the addressed gray level. Fig. 4(a) shows the normalized transmission versus gray level for this configuration ($\varphi_1 = 0$ and $\varphi_2 = -77$), and its complementary ($\varphi_1 = 0$ and $\varphi_2 = -77 + 90$), obtained with wavelength $\lambda = 633$ nm. The curves are identical if $\alpha = +103$ is selected. The brightness control is placed at minimum and the contrast control is placed at maximum. Another video-projector control named color is placed at minimum. Fig. 4(b) presents a numerical simulation of the experiment. In order to compare both figures, the curves in Fig. 4(b) are presented in a descending order with respect to β. This is because if the voltage increases, the value of the birefringence decreases. The range of birefringence is selected between 0 and 1.5π, which matches the experimental range shown in Fig. 4(a). It shows a very good agreement with the experimental curves.

4.3. Adiabatic points obtained by applying voltage. The problem of the surface layers

Another way to locate an adiabatic point is to apply voltage to the LCD. Then the liquid crystal molecules tilt towards the z-axis, and the effective birefringence is changed. If the model by Lu and Saleh is exact, this way permits, for a given wavelength, to change β in the range from $\beta = \beta_{max}$ to $\beta = 0$. However, the LC layers close to the surfaces of the display may not tilt with the voltage, and consequently, the assumptions of the Lu and Saleh

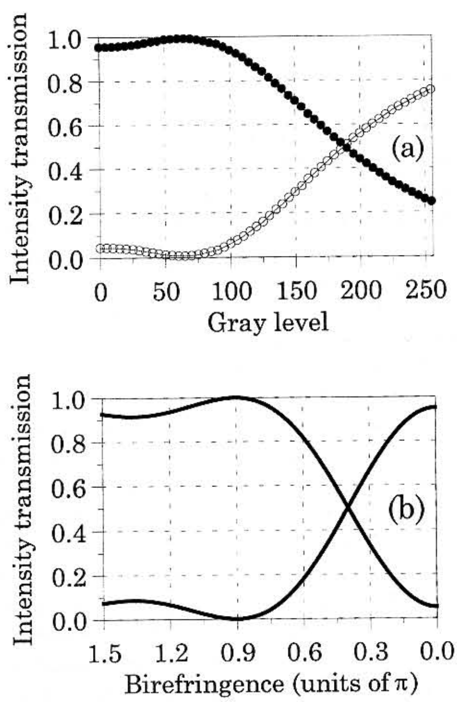

Fig. 4. (a) Experimental normalized transmission for the polarizers configurations $\varphi_1 = 0$ and $\varphi_2 = -77°$ (full circles), and $\varphi_1 = 0$ and $\varphi_2 = -77 + 90°$ (open circles). (b) Theoretical transmission for this configuration as a function of β, and for a LCD with $\alpha = -77°$.

model may fail, and the simple technique used to measure α may not be so useful. This occurs mainly for high values of voltage [14]. In this section, we demonstrate that local adiabatic points can be obtained by applying voltage to the display, but they are affected by these surface layers that do not tilt.

Coy et al. [15] proposed a correction to the Lu and Saleh model to take into account the surface layers. In their model, these layers are considered as two wave plates located at the input and output surfaces of the SLM, and their fast axes are oriented parallel to the LC director at the respective surface. With this model the LCD Jones matrix of Eq. (1) must be modified to a new matrix $\mathbf{M}'_{\mathrm{LCD}}$ given by

$$\mathbf{M}'_{\mathrm{LCD}}(\alpha,\beta,\delta) = \{\mathbf{R}(-\alpha)\mathbf{W}(\delta)\mathbf{R}(+\alpha)\} \times \{\exp(-i\beta)\mathbf{R}(-\alpha)\mathbf{M}(\alpha,\beta)\}\mathbf{W}(\delta), \quad (8)$$

where $\mathbf{W}(\delta)$ represents a wave plate with a fast axis oriented parallel to the x-axis, and introduces a phase shift of value δ, and $\mathbf{M}(\alpha,\beta)$ is the same matrix as in the Lu and Saleh model (Eq. (3)).

When a local adiabatic point is obtained, the matrix $\mathbf{M}(\alpha,\beta)$ becomes an identity matrix and consequently the LCD matrix is now

$$\mathbf{M}'_{\mathrm{LCD}}(\alpha,\gamma=n\pi,\delta) = \exp(-i\beta)\mathbf{R}(-\alpha)\begin{bmatrix}\exp(-i\delta) & 0 \\ 0 & \exp(+i\delta)\end{bmatrix}. \quad (9)$$

In this situation, the action over the light is not only a rotation in the polarization, but also a phase shift between two orthogonal components. This situation is equivalent to the adiabatic regime (Eq. (4)). The twist angle may be obtained by looking for the maximum extinction between polarizers. However, as mentioned earlier, maximum extinction can be obtained even if the LCD is not in a local adiabatic point, if the polarizers are oriented at the appropriate orientations.

In order to demonstrate that local adiabatic points are obtained by applying voltage, we use the experiment shown in Section 4.2. Fig. 4(a) shows maximum transmission equal to one for gray level 65 in the configuration $\varphi_1 = 0$ and $\varphi_2 = \alpha$. According to Eq. (7), $T = 1$ occurs for this configuration only if a local adiabatic point is obtained ($\gamma = m\pi$). Let us note that the intensity transmission for this particular configuration of the polarizers ($\varphi_1 = 0$ and $\varphi_2 = \alpha$), is unaffected by the two wave plates of the model by Coy et al. [15]. Then, the experiment shown in Fig. 4(a) shows that a local adiabatic point is obtained for gray level equal to 65.

In order to see the effect of the voltage on the polarization ellipse, we performed the experiment described in Section 4.1 when a gray level is addressed to the display. Fig. 5 shows the experimental results for the maximum

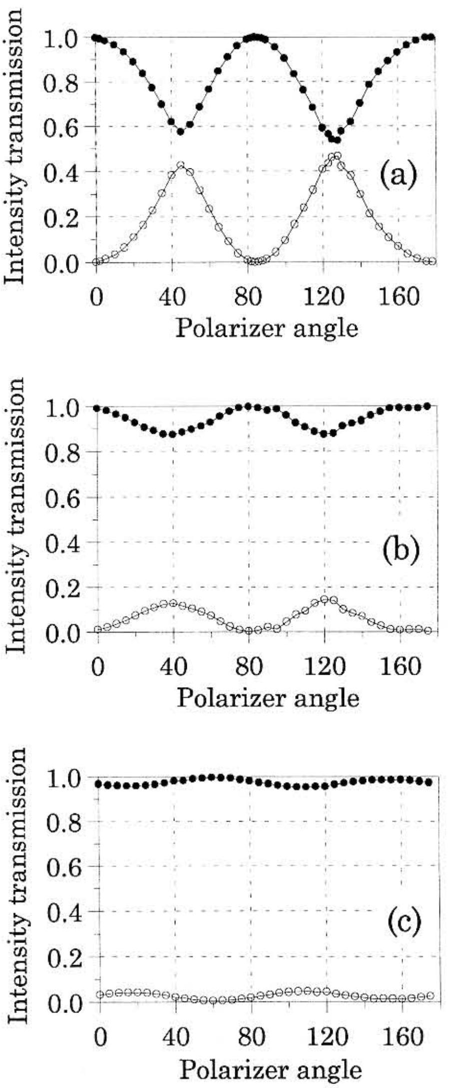

Fig. 5. Experimental results obtained for $\lambda = 633$ nm, for the maximum and minimum transmission versus input polarizer angle. (a) Minimum voltage, (b) medium voltage corresponding to a local adiabatic point, (c) maximum voltage.

and minimum transmission versus input polarizer for three different voltages. Fig. 6 shows the results for the analyzer angle that gives maximum transmission vs. polarizer angle, for the same three voltages. The light wavelength is $\lambda = 633$ nm. Fig. 5(a) and Fig. 6(a) correspond to a minimum voltage (gray level equal to zero), Fig. 5(b) and Fig. 6(b) to the local adiabatic point (gray level equal to 65), and Fig. 5(c) and Fig. 6(c) to a maximum voltage (maximum gray level and brightness and contrast settings also placed to the maximum).

For the first case, the offset voltage applied to the LCD affects the value of β and the local adiabatic regime observed in the off-state for this wavelength (Fig. 3) no

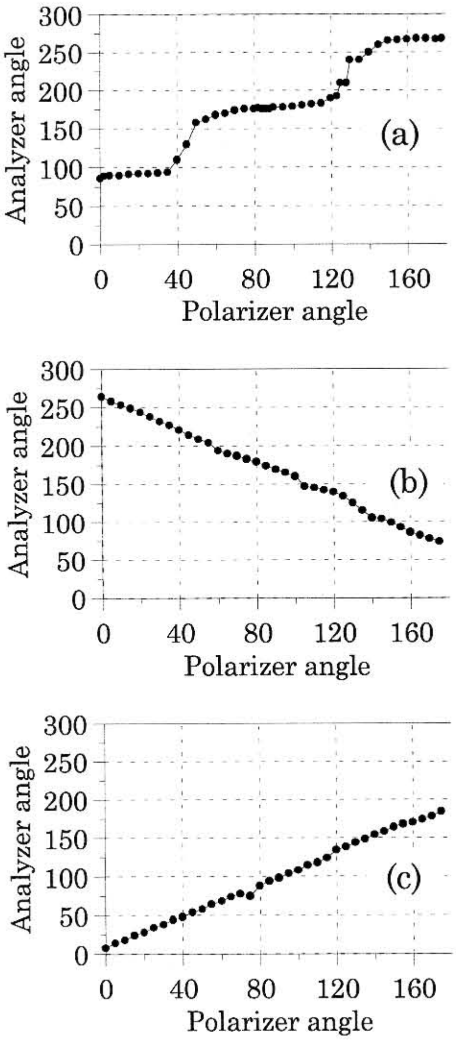

Fig. 6. Experimental results obtained for $\lambda = 633$ nm, for the analyzer angle that gives maximum transmission versus input polarizer angle. (a) Minimum voltage, (b) medium voltage corresponding to a local adiabatic point, (c) maximum voltage.

longer holds. This is observed in the oscillations of the maximum and minimum transmission with respect to the input polarizer angle (Fig. 5(a)). The non-linear curve of Fig. 6(a) confirms non-adiabatic regime.

When a gray level 65 is addressed to the LCD, a local adiabatic point is obtained. However, as Fig. 5(b)Fig. 6(b) show, in this case the behaviour is not the same as in the off-state case. Because of the effect of the surface LC layers, the display produces a phase shift plus a rotation of the polarization. Only for two particular orientations of the incident polarization, maximum extinction is obtained. These orientations coincide with the LC director direction at the input surface ($\psi_D = 80.5$) and the perpendicular orientation. For these two angles of the incident polarization, the light emerges linearly polarized but rotated. The measured rotation on the plane of polarization coincides with the value obtained for the off-state of the LCD, i.e., $\alpha = 103 \pm 1$ or $\alpha = -77 \pm 1°$. Fig. 6(b) shows the orientation of the major axis of the ellipse versus the input angle. In this case, it goes in the opposite sense with respect to the polarizer angle. These results put into evidence that the model of Lu and Saleh [8] breaks when voltage is applied to the LCD. Nevertheless, the approximation of Coy et al. [13] is capable to explain this behaviour.

Finally, we present results when a maximum voltage is applied to the LCD. It is obtained placing all the controls, brightness, contrast and color, at maximum, and addressing a gray level 255 to the display. As shown in Fig. 5(c), the maximum and minimum transmission tend to be constant for all angles of the input polarizer. The relation between the analyzer angle that gives the maximum transmission and the polarizer angle is again linear, but now there is no rotation on the polarization orientation (Fig. 6(c)). This is the expected result for a maximum voltage situation.

5. Determination of the twist sense

We measured the rotation in the plane of polarization when the local adiabatic regime is obtained. However, an ambiguity remains in the sense of the rotation. The measurements show that the twist angle can be either $\alpha = +103°$ or $\alpha = -77°$. To solve this ambiguity, we analyze the rotation induced in the polarization when a high voltage is addressed to the LCD, i.e., for low values of β. Input polarizer is placed at $\varphi_1 = 0$, so maximum transmission is obtained for analyzer placed also at $\varphi_2 = 0$. When the voltage is decreased, β increases and the emerging light is no longer linearly polarized at the input orientation. The angle of the analyzer that gives the maximum transmission will change, and the sense in which it changes depends on the sense of the twist angle. Fig. 7 shows a simulation of the experiment for both cases, $\alpha = +103$

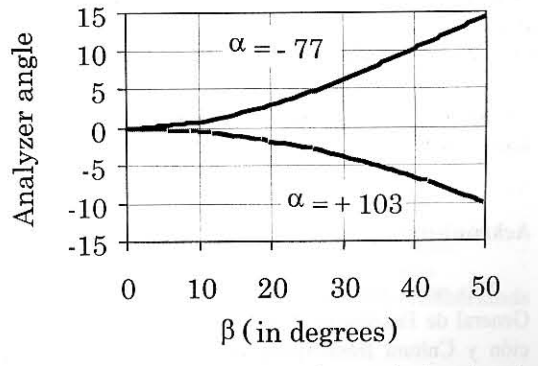

Fig. 7. Theoretical results for the analyzer angle that gives the maximum transmission versus β for $\alpha = 103$ and $\alpha = -77°$.

and $\alpha = -77°$. The angle of the analyzer that gives the maximum transmission is that angle that coincides with the major axis of the polarization ellipse. Note that, as β increases, the polarization ellipse rotates in the opposite sense with respect to the twist angle. We performed this experiment for high values of voltage for our LCD. Because the analyzer angle that gives maximum transmission goes to positive values when β increases, we conclude that the twist angle is negative and consequently, it is $\alpha = -77°$.

6. Conclusions

A method for determining the twist angle in twisted nematic liquid crystal devices has been proposed. We demonstrated, both theoretically and experimentally, that local adiabatic behavior is obtained. In this situation, the twist angle may be determined by measuring the rotation in the polarization plane. Once the twist angle has been measured, then the determination of the other parameters (maximum birefringence and the location of the LC director orientation) may be obtained by other procedures proposed in the literature, which are simplified by the knowledge of the twist angle.

We show that local adiabatic regime can be obtained either by applying voltage to the LCD or by changing the light wavelength. We show that in the off-state local adiabatic behaviour can be obtained by changing the wavelength. This shows that the model by Lu and Saleh [8] is exact when no voltage is applied to the display. Then, the twist angle can be obtained with high accuracy by measuring the rotation in the polarization plane when a local adiabatic point is obtained.

When a voltage is applied, local adiabatic behaviour can be obtained, but it can be affected by the LC layers close to the surfaces that do not tilt. We show that the model of Lu and Saleh [8] breaks down for high values of voltage. We used an approximation that considers the surface layers proposed by Coy et al. [15]. With this approximation, local adiabatic points are equivalent to the adiabatic regime valid for thick LCD. The simple technique proposed to measure α is affected when voltage is applied to the display. Consequently, it is more convenient to use it in the off-state of the display. We give a theoretical explanation of all these situations in terms of the Jones matrix theory.

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References

[1] H.-K. Liu, J.A. Davis, R.A. Lilly, Optical-data-processing properties of a liquid crystal television spatial light modulator, Optics Lett. 10 (1985) 635–637.
[2] I. Moreno, J. Campos, C. Gorecki, M.J. Yzuel, Effects of amplitude and phase mismatching errors in the generation of a kinoform for pattern recognition, Jpn. J. Appl. Phys. 34 (1995) 6423–6432.
[3] H.-K. Liu, T.-H. Chao, Liquid crystal television spatial light modulators, Appl. Optics 28 (1989) 4772–4780.
[4] J.L. Bougrenet de la Tocnaye, L. Dupont, Complex amplitude modulation by use of liquid–crystal spatial light modulators, Appl. Optics 36 (1997) 1730–1741.
[5] J.L. Pezzanitti, R.A. Chipman, Phase-only modulation of a twisted nematic liquid crystal TV by use of the eigenpolarization states, Optics Lett. 18 (1993) 1567–1569.
[6] J.A. Davis, I. Moreno, P. Tsai, Polarization eigenstates for twisted-nematic liquid crystal displays, Appl. Optics 37 (1997) 937–945.
[7] I. Moreno, J.A. Davis, K. D'Nelly, D. Allison, Transmission and phase measurements for polarization eigenvectors in twisted nematic liquid-crystal spatial light modulators, Opt. Eng. 37 (1998) 3048–3052.
[8] K. Lu, B.E.A. Saleh, Theory and design of the liquid crystal TV as an optical spatial phase modulator, Opt. Eng. 29 (1990) 240–246.
[9] C. Soutar, K. Lu, Determination of the physical properties of an arbitrary twisted-nematic liquid crystal cell, Opt. Eng. 8 (1994) 2704–2712.
[10] J.A. Davis, D.B. Allison, K.G. D'Nelly, M.L. Wilson, I. Moreno, Ambiguities in measuring the physical parameters for liquid crystal spatial light modulators, to be published in Opt. Eng.
[11] M. Yamauchi, T. Eiju, Optimization of twisted nematic liquid crystal panels for spatial light phase modulation, Optics Comm. 115 (1995) 19–25.
[12] Y. Zhou, Z. He, S. Sato, A novel method for determining the cell thickness and twist angle of a twisted nematic cell by Stokes parameter measurement, Jpn. J. Appl. Phys. 36 (1997) 2760–2764.
[13] R. Giust, J.-P. Goedgebuer, Determination of the twist angle and the retardation properties of twisted nematic liquid crystal television by spectral measurements, Opt. Eng. 37 (1998) 629–634.
[14] L. Goncalves-Neto, D. Roberge, Y. Sheng, Full-range, continuous, complex modulation by the use of two coupled mode liquid–crystal televisions, Appl. Optics 35 (1996) 4567–4576.
[15] J.A. Coy, M. Zaldarriaga, D.F. Grosz, O.E. Martinez, Characterization of a liquid crystal television as a programmable spatial light modulator, Opt. Eng. 35 (1996) 15–19.
[16] A. Yariv, P. Yeh, Optical Waves in Crystals, Chap. 5, Wiley, New York (1984).
[17] J.C. Kirsch, D.A. Gregory, M.W. Thie, B.K. Jones, Modulation characteristics of the Epson liquid crystal television, Opt. Eng. 5 (1992) 963–970.
[18] V. Laude, S. Mazé, P. Chavel, Ph. Réfrégier, Amplitude and phase coding measurements of a liquid crystal television, Optics Comm. 103 (1993) 33–38.