Modified morphological correlation based on bit-map representations

Amir Shemer, David Mendlovic, Gal Shabtay, Pascuala Garcia-Martinez, and Javier Garcia

Pattern recognition with high discrimination can be achieved with a morphological correlator. A modification of this correlator is carried out by use of a binary slicing process instead of linear thresholding. Although the obtained correlation result is not identical to the conventional morphological correlation, it requires fewer calculations and provides even higher discrimination. Two optical experimental implementations of this modified morphological correlator as well as some experimental results are shown. © 1999 Optical Society of America

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1. Background

Linear correlation is one of the most popular operations in signal processing and in optical signal processing. It can easily be implemented by optical means, e.g., with a VanderLugt 4f correlator¹ or a joint transform correlator² (JTC). Linear correlation is known to be optimal for localizing an object within an input scene in the sense of the meansquared error, defined as³

$$MSE(m) = \sum_{k \in R} \left[f(k+m) - g(k) \right]^2$$

= $\sum_{k \in R} \left[f(k+m)^2 + g(k)^2 - 2f(k+m)g(k) \right],$ (1)

where f and g are the reference and the observed images, respectively. Because $[(f - g)^2 = f^2 + g^2 - 2fg]$ for any two real-valued discrete functions, the mean-squared-error criterion is equivalent to maximizing the linear cross correlation:

$$\gamma_{fg}(m) = \sum_{k \in R} f(k+m)g(k).$$
⁽²⁾

The morphological correlation, on the other hand, is based on minimizing the mean absolute error (MAE), defined as

$$MAE(m) = \sum_{k \in R} |f(k+m) - g(k)|.$$
 (3)

Now using the equation

$$|f - g| = f + g - 2\min(f, g)$$
(4)

allows Eq. (3) to be expressed as

$$MAE(m) = \sum_{k \in R} \{ f(k+m) + g(k) - 2 \min[f(k+m), g(k)] \},$$
(5)

where the sum of f and g in Eq. (5) is a constant value, and therefore minimizing the MAE is equivalent to maximizing the nonlinear cross-correlation expression^{3,4}:

$$\mu_{gf}(m) = \sum_{k} \min[g(k+m), f(k)] = \sum_{q=1}^{Q} g_q(m) * f_q(m),$$
(6)

where the asterisk represents a linear correlation operation, Q represents the maximal gray level of the images, and $g_q(m)$ is the qth binary slice of g(m) obtained by the thresholding of the image according to

$$g_q(m) = \begin{cases} 1 & g(m) \ge q \\ 0 & g(m) < q \end{cases}.$$

$$\tag{7}$$

A. Shemer, D. Mendlovic (mend@tau.eng.ac.il), and G. Shabtay are with the Faculty of Engineering, Tel-Aviv University, Tel Aviv 69978, Israel. P. Garcia-Martinez and J. Garcia are with the Departimento d'Óptica, Universitat de Valencia, Calle Dr. Moliner 50, 46100 Burjassot, Spain.

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Fig. 1. (a) The reference image. (b) The corresponding masks of the reference image.

In this paper we refer to this nonlinear operation [Eq. (6)] as the conventional morphological correlation (CMC). The CMC provides higher discrimination capabilities compared with the linear correlation in pattern-recognition tasks. However, the CMC needs a considerable amount of computational effort to obtain the final result. Therefore it is most desirable to have a method that can give similar results with less computational effort. This situation is what we obtain by using the approach described in Section 2. In Section 2 we present a way to reduce the number of correlated slices by using a binary representation of the gray level of each pixel in an image. This algorithm, which we coin the modified morphological correlation (MMC), reduces the computational complexity of the morphological correlation operation. It also improves the discrimination capabilities with respect to the CMC, but it is not optimal for minimizing the MAE. The discrimination capabilities of the MMC can in some cases be a disadvantage, especially when we deal with additive noise, such as quantization noise, or when slight changes in the intensity of the image or in the background illumination are made.

Recently an optoelectronic implementation for performing the CMC that is based on the above linkage was suggested.⁵ This implementation consists of a JTC for performing the linear correlations and a computer interface for making the threshold decomposition and the required summation operation.

2. Modified Morphological Correlation

We hereby propose an alternative procedure for increasing the selectivity and decreasing the computational requirements of the optoelectronic implementation of the CMC. This method, the MMC, is a modified version of the CMC, and it is not based on linear threshold decomposition but on bitrepresentation decomposition. An effective example of this decomposition is the binary optics process for manufacturing diffractive optical elements.⁶ In these types of decomposition, the image is decomposed into a set of binary slices, each corresponding to a specific bit in the binary representation of the image



Fig. 2. (a) The image masks when the intensity level is 127. (b) The corresponding masks when a unit-value change has occurred.



Fig. 3. JTC scheme: SLM, spatial light modulator; F. T. lens, Fourier transform lens; J. T. plane, joint transform plane.

pixels. For example, an image pixel with gray-level intensity of 100 out of 256 levels will be represented as 01100100 and decomposed into 8 binary slices that have a value of 1 in the seventh, the sixth, and the third slices and a value of 0 in the remaining slices, as shown in Fig. 1. By applying a linear correlation between the binary slice of the reference-image set



Fig. 4. Spatial-multiplexing MMC approach: (a) input-plane arrangement and (b) responses obtained at the output plane.



Fig. 6. Look-up table used for the autocorrelation test (solid curve and the cross-correlation test (dashed curve) with the transformed object.

and the associated slice from the input-scene binary set, one gets

$$\beta_{gf}(m) = \sum_{q=1}^{Q} g_q(m) * f_q(m),$$
(8)

where Q is the number of binary slices. Note that the obtained result is not identical to that of the CMC. An immediate advantage of the MMC is that it saves many of the correlation operations involved in the CMC process. For instance, commonly, a gray-scale image has 256 gray levels. This results in a set of 256 threshold binary slices for calculating the CMC optically.⁵ When the MMC process is applied, one needs only eight binary slices. More generally, for $N = 2^n$ quantization levels the MMC requires $n = \log_2 N$ correlation operations rather than the N operations in the CMC approach.

An additional advantage of the MMC process is that it improves discrimination capabilities and thus yields a more selective system. For explicating this point, let us assume two identical patterns in which we add a single intensity unit to one of the objects. If we assume 256 intensity levels, then in the CMC



Fig. 5. Objects used for the experiments: (a) reference object and (b) observed scene.



Fig. 7. Linear correlation results obtained with the reference object [Fig. 5(a)] and the input scheme [Fig. 5(b)]: (a) captured output correlation plane and (b) profile of the line connecting the two peaks.

case, only one slice is changed, and the total change in the output is approximately 1/256th of the maximal CMC peak value. Thus the discrimination between the two objects is low. On the other hand, in the MMC case, at least one slice is changed, but there are only eight slices. Thus the total change in the output correlation peak is at least one eighth, and the discrimination is significantly better than in the CMC case. For example, if we take an image with a pixel that has a grey level of 127 and we add a unit value to it, this will change all its slices to have a grey level of 128, as can be seen from Fig. 2. However, this example illustrates that the MMC is very sensitive to various kinds of noises (for example, quantiziation noise).

3. Optoelectronic Implementation

The morphological correlation requires a great many calculations; the MMC reduces this number significantly. Nevertheless, the MMC still requires many calculations. This is why optics may be an attractive way to implement the MMC efficiently. Herein, we present two different configurations for implementing the MMC on the basis of time-sequential or spatial-multiplexing concepts.

A. Time-Sequential Concept

The first approach is similar to the one used in Ref. 5. This system is actually a JTC (Fig. 3) that exhibits the cross correlation of two functions by means of a parallel optical computation. Each pair of slices (one slice from the reference object and the other from the input scene) is placed one slice beside the other in the input plane. For each pair optical computation of the joint power spectrum is performed. The summation of the joint power spectrum of these pairs is stored in the computer and finally fed back at the input plane for a second Fourier transformation to produce the MMC. Note that the same setup is also



Fig. 8. Same as Fig. 7 but for the CMC.

capable of implementing the linear correlation and the CMC processes.

B. Spatial-Multiplexing Concept

Whereas the first approach is based on electrical summation of the joint power spectrum, we now propose to take full advantage of the parallelism of optics and to perform the summation optically. We spatially multiplex the thresholded images into a single image to obtain the MMC.

The idea of spatially multiplexing the thresholded images was used by Ochoa *et al.*⁷ for optically implementing median filtering. One can implement the median filter by performing a median filtering in each binary slice and combining the results. Furthermore, a display of red-green-blue (RGB) color images combined into one slice was applied successfully to color pattern recognition by use of a JTC architecture.⁸ To obtain the MMC, we combine the spatial multiplexing of the threshold decomposition and the JTC architecture. We display the four pairs of binary slices used in the time-sequential bit-map approach into one input image. Adequate separation and localization of the scenes is applied, as described in Ref. 8. If the input image of Fig. 4(a) is introduced to the JTC, the final correlation outputs can be calculated. For this configuration we obtain 21 correlation outputs, as shown in Fig. 4(b), which can be written as

$$\begin{split} &C_1 = g_4 * f_1 = C_{21}^*, \\ &C_2 = (g_1 * g_4) + (f_1 * f_4) = C_{20}^*, \\ &C_3 = f_4 * g_1 = C_{19}^*, \\ &C_4 = (g_3 * f_1) + (g_4 * f_2) = C_{18}^*, \\ &C_5 = (g_1 * g_3) + (g_2 * g_4) + (f_1 * f_3) + (f_2 * f_4) = C_{17}^*, \end{split}$$



Fig. 9. Same as Fig. 7 but for the MMC.

$$\begin{split} C_6 &= (f_3 * g_1) + (f_4 * g_2) = C_{16}^*, \\ C_7 &= (g_2 * f_1) + (g_3 * f_2) + (g_4 * f_3) = C_{15}^*, \\ C_8 &= (g_1 * g_2) + (g_2 * g_3) + (g_3 * g_4) + (f_1 * f_2) \\ &+ (f_2 * f_3) + (f_3 * f_4) = C_{14}^*, \\ C_9 &= (f_2 * g_1) + (f_3 * g_2) + (f_4 * g_3) = C_{13}^*, \\ C_{10} &= (g_1 * f_1) + (g_2 * f_2) + (g_3 * f_3) + (g_4 * f_4) = C_{12}^*, \\ C_{11} &= \text{Addition of all autocorrelation terms.} \end{split}$$

Taking into account the last 2 equations, we see that the term C_{10} is the MMC as defined in Eq. (8). Thus when spatial multiplexing is feasible the MMC can be obtained in a single step. Extension to a higher number of slices is conceptually trivial. Nevertheless, it requires a higher-spatial-resolution display. The MMC will be located along the x axis at the first diffraction order if the same configuration is used.

4. Experimental Results and Computer Simulations

The optoelectronic system, which is shown in Fig. 3, was constructed to test the performance of the MMC in both the time-sequential and the spatialmultiplexing concepts. Figure 5 shows the reference object [Fig. 5(a)] that is to be detected within the observed scene [Fig. 5(b)]. Both the reference object and the observed scene are presented through 16 gray levels. Thus, to perform the CMC between them, one needs to threshold and correlate 16 binary slices, whereas only four correlations are used for performing the MMC that is based on binary representation of those images (or only one correlation if the spatial-multiplexing approach is taken).

The observed image consists of the original refer-



Fig. 10. Simulation results for the MMC by use of the spatialmultiplexing approach.

ence object and another similar object for testing both the autocorrelation and the cross-correlation performance. The false object shown in Fig. 5(b) is actually the reference object wherein the intensity was transformed according to the look-up table shown in Fig. 6. Using the time-sequential concept, we show in Figs. 7(a), 8(a), and 9(a) the obtained output intensities after performing the linear correlation, the CMC, and the MMC, respectively. The corresponding cross sections of the intensity-peak regions are displayed in Figs. 7(b), 8(b), and 9(b) with a linear scale. Note that these schemes include the lower part of the JTC system's output plane, which consists of the resultant zeroth and first diffraction orders. All correlation processes provide sharp correlation peaks for the original reference input. The results clearly indicate that the linear correlation yields a false detection for the transformed image input, whereas the CMC and the MMC yield a low response (48% for the CMC and 27% for the MMC) for the false image input (Figs. 8 and 9, respectively).

In addition, computer simulations were carried out

to demonstrate the spatial-multiplexing concept. Figure 10 shows the obtained result (C_{10}) for the same reference and input as for Fig. 5. Note that this approach becomes more complex when the number of slices needed for the image increases and that the result shown in Fig. 10 consists of only the MMC part (C_{10}) of the overall resultant correlation output scheme.

5. Conclusions

In conclusion, this paper has discussed a variation of the morphological correlation, coined the modified morphological correlation (MMC), and its possible optical implementations. The MMC was found to be highly discriminative and to have lower computational complexity in comparison with the CMC. Furthermore, we have also demonstrated two systems for optically implementing the MMC. One is based on multiple cycles (time sequential) and the other on a single cycle (spatial multiplexing) of the JTC system. Experimental results that demonstrate the capabilities of the MMC are provided.

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