Shift- and scale-invariant recognition of contour objects with logarithmic radial harmonic filters

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The phase-only logarithmic radial harmonic (LRH) filter has been shown to be suitable for scale-invariant block object recognition. However, an important set of objects is the collection of contour functions that results from a digital edge extraction of the original block objects. These contour functions have a constant width that is independent of the scale of the original object. Therefore, since the energy of the contour objects decreases more slowly with the scale factor than does the energy of the block objects, the phase-only LRH filter has difficulties in the recognition tasks when these contour objects are used. We propose a modified LRH filter that permits the realization of a shift- and scale-invariant optical recognition of contour objects. The modified LRH filter is a complex filter that compensates the energy variation resulting from the scaling of contour objects. Optical results validate the theory and show the utility of the newly proposed method. © 2000 Optical Society of America

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1. Introduction

In recent years one of the main trends in optics has been pattern recognition, owing to the ability of optical systems to perform a correlation in real time. Most of the optical implementations are based either on the VanderLugt correlator[1] (by means of filtering in the Fourier domain) or on the joint transform correlator[2] (by means of processing in the image plane). In both cases the method is based on obtaining the correlation between the input scene, which contains the object to be detected, and the target or a function related to it. The correlation provides a measure of similarity connected with the mean-squared difference between the patterns to be correlated. However, correlation has one main drawback: high sensitivity to deformations of the object to be detected.

In many applications it would be desirable to find methods that could provide some distortion-invariant recognition. Typically two deformations are considered: scale and rotation changes of the target. Several methods have been proposed for obtaining different kinds of distortion invariance, most of them based on the use of filters partially matched to the target.

In a first attempt Casasent and Psaltis[3,4] obtained scale and rotation invariance with a method based on the Mellin transform but at the price of loosing the shift invariance. Later, Duvernoy,[5] describing the optical Fourier spectra with statistical descriptors, Sheng and Duvernoy,[6] Sheng and Lejeune,[7] and Sheng and Arsenault,[8] using circular-Fourier–radial-Mellin transform descriptors, obtained invariance to translation, rotation, and scale; the recognition of the target was implemented as a classification in a multidimensional feature space. More recently the recognition with these three main invariances was obtained by Fang and Häusler,[9] using an original transformation.

Another different approach to the problem is based on the decomposition of the function representing the target into orthogonal components. Instead of modifying the input image, several modifications of the matched filter are introduced. So the rotation problem has been overcome with the circular-harmonic expansion.[10] A single circular-harmonic component (CHC) is used as impulse response of the matched filter,[11,12] providing shift and rotation invariance in the correlation plane. Since each CHC is multiplied by a function $\exp(im\alpha)$ when the object to be detected rotates an angle α (m being the order of the CHC), the

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intensity of the correlation peak does not change if a single CHC is used in the filter plane.

The scale-invariance problem is more complicated than the rotation one, because rotation is a periodic function, whereas scale may in principle change without any limit. Thus complete rotation invariance is possible, whereas scale invariance must be limited to a given range. Apart from the methods concerning transformations of the input plane, much research has been done with different kinds of filters. Szoplik[13] and Szoplik and Arsenault[14] obtained partial scale invariance in an anamorphic Fourier correlator and with multiple circular-harmonic filters. To overcome some difficulties of the previous filters, coordinate-transformed phase-only filters have been proposed.[15,16] Unlike anamorphic filters, for which a given frequency (u, v) changes its radial and angular positions with respect to a symmetrical system, with the filters proposed in Refs. 15 and 16 only the radial position shifts, keeping the same angular position.[15] The radial stretching of the filters depends on the cumulative angular distribution of the energy of the target spectrum.[16] These filters are partially matched to the object in different angular sectors of the filter. It is also possible to obtain a match for different scales by use of the scaling introduced by different wavelengths.[17,18]

In a way analogous to the circular-harmonic decomposition method used for obtaining shift- and rotation-invariant recognition, Mendlovic *et al.*[19] applied an orthogonal decomposition of the target into Mellin radial harmonics to obtain shift and scale invariance. The filter is matched to a single component of the expansion. The scale invariance, however, should be interpreted in the sense that the correlation intensity distribution is scaled with the same factor as the input pattern. For block objects the output intensity is not strictly invariant but depends quadratically on the scale factor of the object.

The variation of the correlation-peak intensity with the scale factor can be avoided in part by means of employing a filter derived from the previous idea and proposed by Rosen and Shamir.[20] Using an expansion analogous to the Mellin radial harmonics, although not orthogonal, into logarithmic radial harmonics (LRH) in the Fourier plane, they proposed a phase filter that provided good results in a broad range of scales. Moya *et al.*[21] extended this study to obtain projection-invariant (one-dimensional scaling) pattern recognition with a phase-only logarithmic-harmonic-derived filter with good performance.

Up to now, in all these methods, the input objects were conventional two-dimensional block objects. Nevertheless, an important set of objects is the one that results, for instance, from an edge extraction of the original block objects (see, for instance, Refs. 22–24). This collection is formed by the contour functions, which have a constant width that is independent of the scale of the original object. Thus it can be expected that the behavior under the scale-invariance problem of this type of object will be different from the one for block objects described, for instance, in Refs. 4, 16, 17, 19, and 20. The contour images resulting from edge detection can be described and analyzed digitally, for example, by chain-code methods (see Ref. 25, for instance). Optical methods, however, provide an opportunity for parallel and real-time processing.

Therefore in this paper we introduce a new approach to the scale-invariance problem for contour objects, using the LRH expansion. In Section 2 we review the main features of the scale-invariant optical correlators, using LRH decomposition. In Section 3 we extend the previously reported theory to the case of contour objects. In Section 4 experimental results show the utility of the newly proposed technique, and in Section 5 the main conclusions are outlined.

2. Scale Invariance with Logarithmic Radial Harmonics Decomposition

Given an input object function $f(r, \theta)$ and its scaled version with factor β, $f(\beta r, \theta)$ (both expressed in polar coordinates, considering the same origin for simplicity), the relation between their Fourier transforms (FT's) can be written as

$$G(\rho, \phi) = \frac{1}{\beta^2} F\left(\frac{\rho}{\beta}, \phi\right), \tag{1}$$

$F(\rho, \phi)$ and $G(\rho, \phi)$ being the FT of the functions $f(r, \theta)$ and $f(\beta r, \theta)$, respectively. This equation indicates that the FT of a scaled function is proportional to the scaled FT of the original function. Rosen and Shamir[20] used this property to define a new filter in the Fourier plane of an optical correlator, which allowed for scale-invariant pattern recognition. The general structure of the filter is $H(\rho, \phi) = R(\rho)S(\phi)$. Considering the input object function $f(r, \theta)$ in an optical system with this filter in its Fourier plane, the value of the obtained correlation center can be written as

$$C_{f,h} = \int_d^D \int_0^{2\pi} F(\rho, \phi) R^*(\rho) S^*(\phi) \rho \mathrm{d}\rho \mathrm{d}\phi, \tag{2}$$

where D is the maximum radius of the filter, d is the radius of a high-pass filter, and the asterisk denotes a complex conjugate. If we consider the input object function as a scaled version $f(\beta r, \theta)$ of the original one, the correlation center value can be expressed as

$$C_{f,h}^{\beta} = \int_{d/\beta}^{D/\beta} \int_0^{2\pi} F(\tau, \phi) R^*(\beta\tau) S^*(\phi) \tau \mathrm{d}\tau \mathrm{d}\phi, \tag{3}$$

where the parameter τ denotes ρ/β.

To have a scale-invariant filter, the relation between Eqs. (2) and (3) must be

$$C_{f,h}^{\beta} = C_{f,h} \exp[i\sigma(\beta)], \tag{4}$$

$\sigma(\beta)$ being a real function depending only on the scale factor β. This condition allows us to define the scale-invariant filter, known as the phase-only LRH filter, as[20]

$$H^*(\rho, \phi) = \exp[i\Omega(\phi)](\rho/d)^{i(p/w)}, \tag{5}$$

where p is the LRH frequency, w is a normalization constant defined by

$$w = \frac{1}{2\pi} \ln\left(\frac{D}{d}\right), \tag{6}$$

and $\Omega(\phi)$ is an angular phase function that carries all the angular information contained in the phase of the object function

$$\Omega(\phi) = -\arg\left[\int_d^D F(\rho, \phi)\left(\frac{\rho}{d}\right)^{i(p/w)} \rho d\rho\right]. \tag{7}$$

Thus the correlation value $C_{f,h}^{\beta}$ yields

$$C_{f,h;p}^{\beta} = \left(\frac{\beta}{d}\right)^{i(p/w)} \int_0^{2\pi} \exp[i\Omega(\phi)] \times \left[\int_{d/\beta}^{D/\beta} F(\tau, \phi)\tau^{i(p/w)}\tau d\tau\right] d\phi, \tag{8}$$

and in the particular case in which the scale factor $\beta = 1$:

$$|C_{f,h;p}^{1}| = \int_0^{2\pi} \left|\int_d^D F(\tau, \phi)\tau^{i(p/w)}\tau d\tau\right| d\phi. \tag{9}$$

As we can see by comparing Eqs. (8) and (9), expression (8) exactly satisfies relation (4) only when the scale factor $\beta = 1$. In any other case, as explained in Ref. 20, relation (4) is accomplished approximately for a certain scale range because of the β dependence on the integration limits.

3. Logarithmic Radial Harmonics Decomposition for Contour Objects

As stated above, an important set of objects is the collection of contour functions that results from a digital edge extraction of the original objects. These contour functions have a constant width that is independent of the scale of the original object. Therefore the energy of the contour objects decreases more slowly with the scale factor than the energy of the block objects. Thus this type of object does not satisfy Eq. (1). To obtain the relation equivalent to Eq. (1) for contour objects, let us consider Fig. 1, in which for the sake of simplicity we have considered two circular objects. The object in Fig. 1(a), which represents a circular crown of average radius r_0 and width Δ, is different from zero (and equal to a real constant A) only when $r \in [r_0 - (\Delta/2), r_0 + (\Delta/2)]$. Its FT can be easily calculated as

$$F(\rho, \phi) = A \int_{r_0-\Delta/2}^{r_0+\Delta/2} \int_0^{2\pi} \exp[-i2\pi r\rho \cos(\phi - \theta)] r dr d\theta. \tag{10}$$

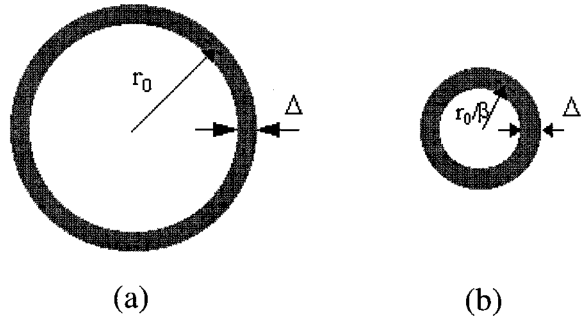

Fig. 1. Circular crown of average radius r_0 and width Δ: (a) without scaling, (b) with scale factor β.

Analogously, the object in Fig. 1(b), which is a scaled version with factor β of the previous object, is different from zero only when $r \in [(r_0/\beta) - (\Delta/2), (r_0/\beta) + (\Delta/2)]$. Now, its FT can be written as

$$G(\rho, \phi) = A \int_{r_0/\beta-\Delta/2}^{r_0/\beta+\Delta/2} \int_0^{2\pi} \exp[-i2\pi r\rho \times \cos(\phi - \theta)] r dr d\theta. \tag{11}$$

Considering that usually $r_0 \gg \Delta$, we can obtain an accurate relation between the FT of the original object and the FT of its scaled version as

$$G(\rho, \phi) = \frac{1}{\beta} F\left(\frac{\rho}{\beta}, \phi\right). \tag{12}$$

This relation, although not completely exact, satisfies the real case better than the relation in Eq. (1). For a general contour object, without any circular symmetry, the result in Eq. (12) holds as well, because the only difference is that now the value of r_0 in the integration limits of Eq. (10) is dependent on the azimuthal angle θ. In this case the minimum value of $r_0(\theta)$, $r_0(\theta)_{\min}$, must fulfill $r_0(\theta)_{\min} \gg \Delta$. To test this property experimentally, we consider objects A1 and A2 shown in Fig. 2. We digitally calculated the modulus of their FT, and in Fig. 3 we represent their horizontal profiles with a curve that contains the central maximum. In this figure we can see the performance given by the relation 12. Specifically, the central maximum values for the FT of the objects are 82,620 arbitrary units for A1 and 39,270 arbitrary units for A2. These values approximately fulfill relation (12), considering that the scale factor between objects A1 and A2 is $\beta = 2$.

Assuming the relation given in Eq. (12), we can calculate the value of the obtained correlation center when we use contour objects

$$C_{f,h}^{\beta} = \beta \int_{D/\beta}^{D/\beta} \int_0^{2\pi} F(\tau, \phi) R^*(\beta\tau) S^*(\phi) \tau d\tau d\phi. \tag{13}$$

Once more the presence of the scale factor β in this equation impedes the LRH filter from being scale

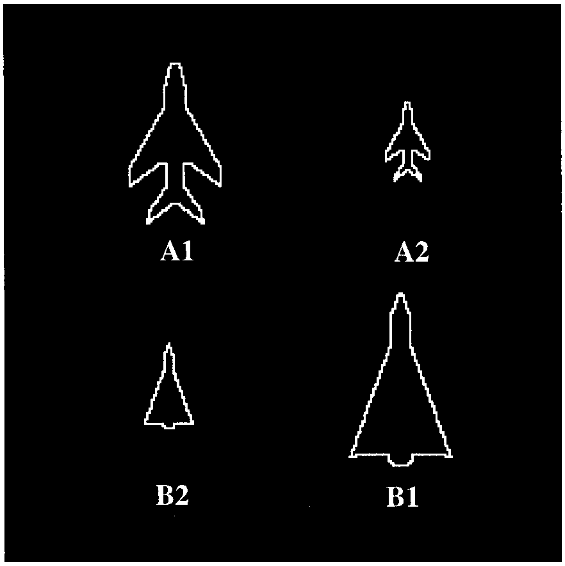

Fig. 2. Contour functions used as input objects in experiments. Objects A2 and B2 are two scaled versions with scale factor $\beta = 2$ of objects A1 and B1, respectively.

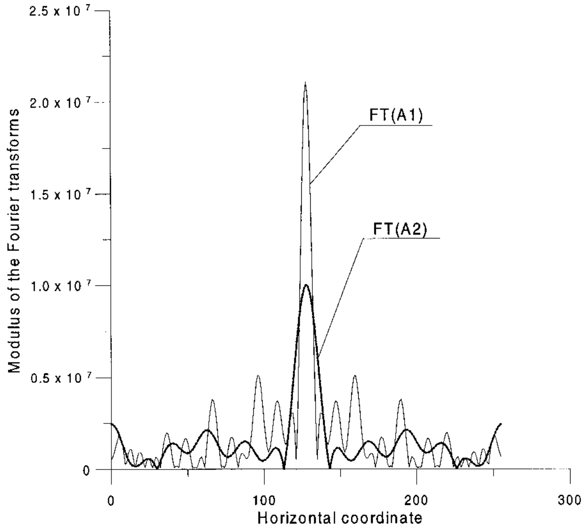

Fig. 3. Horizontal profile through a curve that contains the central maximum of the modulus of the Fourier transform for objects A1 and A2.

invariant, because the intensity of the correlation peak depends on the scale factor.

Thus, to retrieve the ideal performance of the filter [see Eq. (4)], we define a new filter, which will be valid for contour objects. This new modified LRH (MLRH) filter has the form

$$H^*(\rho, \phi) = \exp[i\Omega(\phi)](\rho/d)^{i(p/w)-1}, \tag{14}$$

p, w, and $\Omega(\phi)$ being the functions previously defined. Now, the filter leaves its only phase performance. Introducing this filter in Eq. (13), we can easily obtain

$$C_{f,h;p}^{\beta} = \left(\frac{\beta}{d}\right)^{i(p/w)} d \int_0^{2\pi} \exp[i\Omega(\phi)] \times \left[\int_{d/\beta}^{D/\beta} F(\tau, \phi)\tau^{i(p/w)} \mathrm{d}\tau \mathrm{d}\phi\right], \tag{15}$$

and in the particular case in which the scale factor $\beta = 1$:

$$|C_{f,h;p}^{1}| = d \int_0^{2\pi} \left| \int_d^D F(\tau, \phi)\tau^{i(p/w)} \mathrm{d}\tau \right| \mathrm{d}\phi. \tag{16}$$

Thus we have eliminated the dependence on the scale factor β in the intensity of the correlation peak, but still it remains a dependence on the integration limits. The behavior of the new filter defined for contour objects is similar to the filter for block objects proposed by Rosen and Shamir.[20]

4. Simulated and Optical Results

We used the contour objects shown in Fig. 2. With these contour objects we digitally tested the behavior of the phase-only LRH filter and the MLRH filter. First, we made the correlation between the phase-only LRH filter matched to object A2 with objects A1 and A2. We used the values $D = 70$ pixels and $d = 5$ pixels. In Fig. 4 we represent the intensity of the correlation centers in terms of the frequency of the LRH filter. Second, we made the same correlation but with the MLRH filter matched to object A2. The results for this case are represented in Fig. 5. Comparing Figs. 4 and 5, we can see the scale-invariant property present in the case of the MLRH filter. Because the intensity of the two correlations in Fig. 5 coincide for the frequency filter $p = 2.3$, we used this value to make the filters. To prove the discrimination capability of the newly proposed MLRH filter, we digitally made the correlation of the phase-only LRH

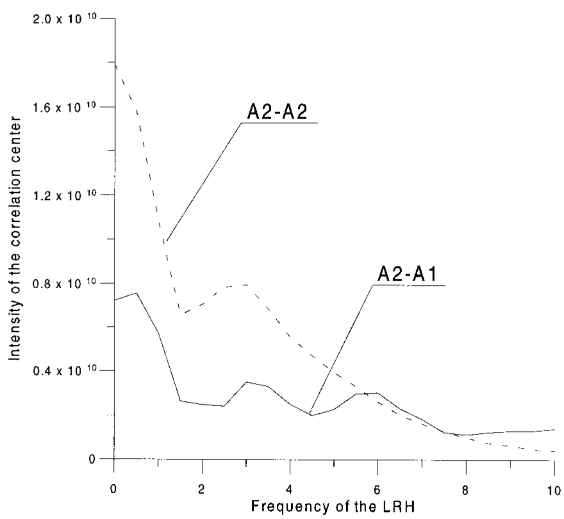

Fig. 4. Intensity of correlation centers in terms of frequency p of phase-only LRH filter matched to object A2 during correlation with object A1 (A2–A1) and object A2 (A2–A2).

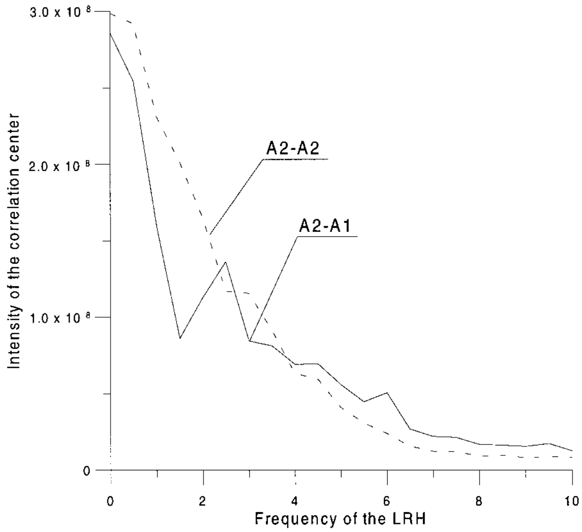

Fig. 5. Intensity of the correlation centers in terms of frequency p of MLRH filter matched to object A2 during correlation with object A1 (A2–A1) and object A2 (A2–A2).

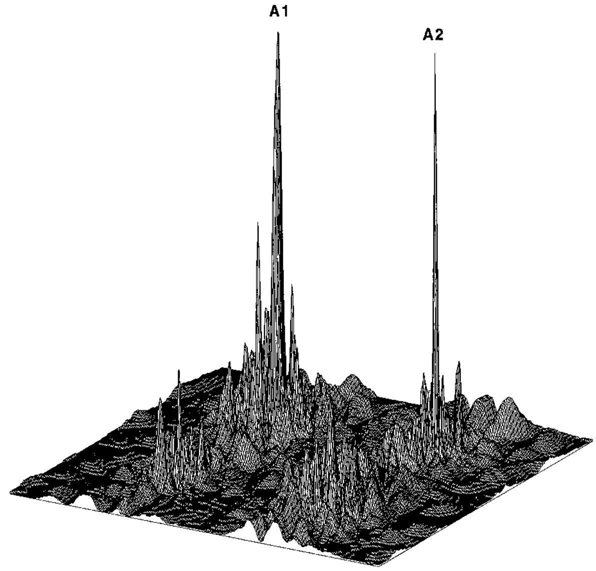

Fig. 7. Simulated correlation between input scene shown in Fig. 2 and MLRH filter matched to object A2 with frequency $p = 2.3$.

filter and the MLRH filter (both matched to object A2 and with $p = 2.3$) with the complete scene shown in Fig. 2. The correlation planes are shown in Fig. 6 (for the phase-only LRH filter) and in Fig. 7 (for the MLRH filter). These results reveal both the discrimination capability of the filters and the scale-invariant property in the case of the MLRH filter.

Finally, using a classical convergent correlator, we optically obtained these last correlations. To do that, the scene shown in Fig. 2 and the filters generated by computer (calculated with the Lohmann detour phase method[26] in 256×256 cells with a resolution of 17×17 pixels/cell and plotted with a 600-dot/in. laser printer) were photoreduced[27] on a lithographic film. The final correlation plane was captured with a Pulnix Model TM-765 CCD camera. The results obtained are shown in Fig. 8 (for the phase-only LRH filter) and in Fig. 9 (for the MLRH filter). For this last case, since the MLRH filter has left its phase-only performance, the background noise level is greater than in the phase-only LRH filter case. It is important to note that the correlation peak that corresponds to the detection of object A1 in Fig. 9 is greater than we could expect. This is due to relation (12), which overloads the energy of object A2 when in fact its energy is smaller. Even so, these optical results confirm those obtained digitally and validate the newly proposed approach.

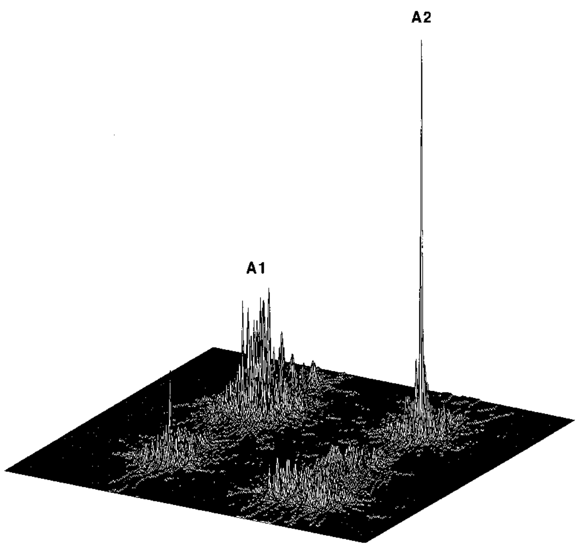

Fig. 6. Simulated correlation between input scene shown in Fig. 2 and phase-only LRH filter matched to object A2 with frequency $p = 2.3$.

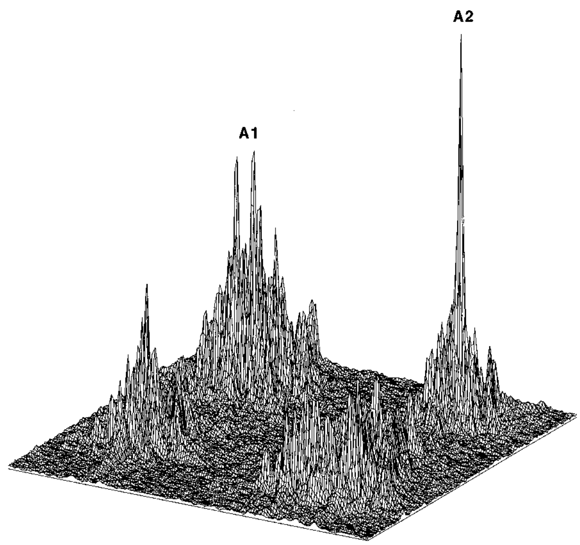

Fig. 8. Optical correlation between input scene shown in Fig. 2 and phase-only LRH filter matched to object A2 with frequency $p = 2.3$.

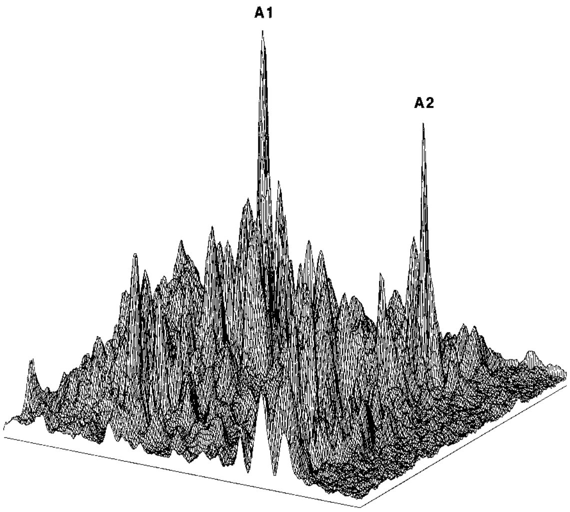

Fig. 9. Optical correlation between input scene shown in Fig. 2 and MLRH filter matched to object A2 with frequency $p = 2.3$.

5. Conclusions

The phase-only LRH filter is suitable for scale-invariant block object recognition. Nevertheless, an important set of objects is that formed by the contour objects resulting, for instance, from an edge-extraction operation. These objects have a constant width that is independent of the scale of the original object. As a consequence, since the energy of the contour objects diminishes more slowly with the scale factor than does the energy of the block objects, the phase-only LRH filter presents some difficulties in pattern-recognition tasks with this type of object. Thus we have proposed a modified LRH filter that permits the realization of a scale-invariant optical correlator for contour objects. The modified LRH filter is a complex filter that compensates the energy variation during scaling of contour objects. Even so, this compensation is approximate, because we assume that the relation given in Eq. (12), which overloads the energy of the object when in fact its value is smaller. However, optical results validate the theory and show the utility of the newly proposed method.

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