# Recognition of unsegmented targets invariant under transformations of intensity

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Images taken in noncooperative environments do not always have targets under the same illumination conditions. There is a need for methods to detect targets independently of the illumination. We propose a technique that yields correlation peaks that are invariant under a linear intensity transformation of object intensity. The new locally adaptive contrast-invariant filter accomplishes this by combining three correlations in a nonlinear way. This method is not only intensity invariant but also has good discrimination and resistance to noise. We present simulation results for various intensity transformations with and without random and correlated noise. When the noise is high enough to threaten errors, the method trades off intensity invariance in order to achieve the optimum signal to noise ratio, and the peak to sidelobe ratio in the presence of clutter is always greater than one. In the presence of random disjoint noise, the signal to noise ratio is independent of the target contrast and of the level of the noise. © 2002 Optical Society of America

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### Introduction

Image recognition of unsegmented targets in a noncooperative environment is a difficult task. When the illumination within a scene cannot be controlled, the outputs for similar targets can be quite different. For example, the matched filter yields an output that is proportional to the target intensity. So there is a need for methods that can detect targets under intensity variations. Many pattern recognition techniques use the correlation operation to detect unsegmented targets. But correlation is a linear operation, which mean that if we multiply a target by an unknown constant factor, the correlation peak height will change by the same amount. Because detection is often determined by means of a threshold on the correlation plane, dark objects can be missed. However, high intensity image clutter can cause false alarms.

One way to achieve intensity invariant recognition with the correlation is to perform a preprocessing on

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images before attempting recognition or to combine multiple correlations.

In this paper we consider two types of intensity transformations: multiplicative and additive. The former occurs when the light source intensity changes, thus affecting in the same way the light reflected from the object. Additive intensity transformations may occur, for example, when different camera settings are used or when scattered light enters the camera. That may introduce a constant difference between the intensity distributions for two identical objects. All the intensity transformations considered here are uniform, i.e., every part of an object is affected by the same intensity transformation.

Dickey and Romero<sup>1,2</sup> introduced a method using the Cauchy-Schwarz inequality to solve this problem: The method yields correlation peak values that are invariant under a multiplicative factor. Kotynski and Chalasinska<sup>3,4</sup> generalized this algorithm by using the Hölder inequality, and ended up with a whole new family of solutions of which the latter is a special case. Arsenault  $et \ al.^5$  proved that it is possible to achieve additive intensity invariance using only one correlation. They used a composite<sup>6</sup> or synthetic discriminant filter<sup>7</sup> constructed to recognize the target but to discriminate against the target support (a binary image equal to unity, where the target is present and equal to zero everywhere else). Arsenault and Belisle<sup>8</sup> also developed a method invariant to changes of orientation and multiplicative

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intensity by combining multiple correlation planes resulting from correlations with different circular harmonic components.<sup>9</sup> Garcia-Martinez et al.<sup>10</sup> proposed a method to achieve intensity-invariant recognition using the sliced orthogonal nonlinear generalized correlation.<sup>11,12</sup> Recently, Arsenault and Lefebvre<sup>13</sup> used a homomorphic transformation to change a multiplicative-intensity problem to an additive-intensity problem that can be addressed with the synthetic discriminant filter mentioned above. Zhang and Karim<sup>14</sup> pointed out that the morphological correlation is intensity invariant when object is brighter than the reference. the Chalasinska-Macukow et al.<sup>15,16</sup> used pure phase correlation to achieve linear-intensity invariance, but this only works well for segmented images. Rahmati *et al.*<sup>17</sup> invented a method that is distortion and intensity invariant. It is also possible to alleviate the problem of an additive constant with linear filtering by setting the filter zero-order frequency to zero.18

### **Intensity Transformations**

A linear transformation of intensity over a target can be expressed as

$$f'(x) = af(x) + b\delta(x), \tag{1}$$

where  $\Diamond(x)$  is the binary object support which is equal to unity over the support of target f(x), and equal to zero everywhere else, and a, b are unknown constants—in other words  $\Diamond(x)$  is a binary version of f(x), so it has the same silhouette as the reference f(x) and is equal to zero outside of this contour. We use one-dimensional notation for simplicity, but the results are easily generalized to two dimensions.

### **Intensity Invariant Recognition**

From previous work done by Arsenault *et al.*<sup>5</sup> it is possible to obtain a correlation plane invariant against a constant additive factor. This can be accomplished by means of an synthetic discriminant function filter h(x) that recognizes the object f(x), but that discriminates against the object support  $\Diamond(x)$ . This filter was originally conceived to detect objects in the presence of non-zero-mean additive noise. The additive contribution for the case of Eq. (1) can be seen as additive noise with zero variance. Using the composite filter theory<sup>6,7</sup> we have the following constraints:

$$h(x) = \alpha f(x) + \beta \delta(x),$$
  
[  $f(x) * h(x)$ ]<sub>x=0</sub> = 1,  
[ $\delta(x) * h(x)$ ]<sub>x=0</sub> = 0, (2)

Although  $\alpha$  and  $\beta$  can be determined by means of numerical techniques, this system can be solved analytically:

$$lpha = rac{R_{\diamond\diamond}}{R_{ff}R_{\diamond\diamond}-R_{f\diamond}^{2}},$$

$$\beta = -\frac{R_{\Diamond f}}{R_{ff}R_{\diamond \Diamond} - R_{f\circ}^{2}},\tag{3}$$

where

$$R_{ff} = [f(x) * f(x)]_{x=0},$$
  

$$R_{\diamond\diamond} = [\diamond(x) * \diamond(x)]_{x=0},$$
  

$$R_{f\diamond} = R_{\diamond f} = [f(x) * \diamond(x)]_{x=0}.$$
(4)

If we substitute those relations in the expression for h(x) we obtain

$$h(x) = \frac{R_{\Diamond \Diamond}}{R_{ff}R_{\diamond \circ} - R_{f^{\diamond}}} \left[ f(x) - \frac{R_{\diamond f}}{R_{\diamond \circ}} \, \Diamond(x) \right].$$
(5)

The fraction in front of  $\Diamond(x)$  is the mean value  $f_0$  of f(x) that yields:

$$h(x) \propto f(x) - \mu_f \Diamond(x) = f_0(x), \tag{6}$$

where  $\mu_f$  is the mean of f(x). The filter h(x) is not only invariant to an additive constant but is also very discriminating.

This filter yields correlation plane values that are proportional to the multiplicative constant a. To eliminate that factor we divide the value of each point of the correlation plane by a value proportional to the multiplicative factor a. We propose to use the variance of the pixel values within the object support  $\delta(x)$ for each point of the scene. This value will not depend on b because the variance is calculated about the mean and will be proportional to  $a^2$ . The local variance for each point over the scene s(x) is equal to

$$\frac{\left[s^{2}(x)*\Diamond(x)\right]}{N} - \frac{\left[s(x)*\Diamond(x)\right]^{2}}{N^{2}}.$$
 (7)

Where N is the number of pixels in the support of the object. Normalizing the zero mean filter correlation plane by means of the variance leads to the final expression for the locally adaptive contrast-invariant filter (LACIF):

$$C = \frac{1}{(f_0 * f_0)_{x=0}} \times \frac{[s(x) * f_0(x)]^2}{[s^2(x) * \Diamond(x)] - 1/N[s(x) * \Diamond(x)]^2}.$$
 (8)

We divided the expression by  $N(f_0(x) * f_0(x))$  to normalize the autocorrelation to one. In sum, the value of invariant parameter *C* can be computed from three correlations combined in a simple way. Note that although the above expression implies a kind of locally adaptive processing, the operations carried out involve only the calculation of three correlations and the calculation of their ratio for each point of the output plane. No segmentation or identification of regions of interest is involved.

## Vector Interpretation for the Locally Adaptive Contrast-Invariant Filter

In the previous section we showed how to construct a recognition parameter that is invariant to the linear intensity transformation of Eq. (1). We now obtain the same expression from a vector space analysis.

As before, the intensity invariance problem that we are trying to solve is to determine if an object is a linear combination of the two components f(x) and  $\delta(x)$  of Eq. (1), that is, whether or not an unknown object g(x) belongs to the subspace spanned by f(x) and  $\delta(x)$ . First, we need to find an orthogonal basis for the subspace, which is a two-dimensional plane. If we choose  $\delta(x)$  to be one of these two components, it is clear that the other orthogonal component must be proportional to  $f_0(x)$ . The LACIF technique is equivalent to projecting g(x) onto the subspace orthogonal to  $\delta(x)$  to obtain  $g_0(x)$ , and to calculate the cosine of the angle between  $g_0(x)$  and  $f_0(x)$ .

If g(x) is truly in the subspace, then the projection yields a component proportional to  $f_0(x)$ . And because the cosine of the angle between two colinear unit vectors is always equal to one, the result will also be equal to one. However, if g(x) is not in the subspace after the projection,  $g_0(x)$  may have a  $f_0(x)$ component but will also have another component. So the angle between  $g_0(x)$  and  $f_0(x)$  is greater than zero, and its cosine is smaller than one. Mathematically speaking, we want to compute the relationship:

$$\cos^2 \theta = \frac{[g_0(x) \circ f_0(x)]^2}{|g_0(x)|^2 |f_0(x)|^2}.$$
(9)

But how can we compute this quantity when all we know is g(x) and when we are dealing with unsegmented objects? In the numerator,  $g_0(x)$  can be replaced by g(x) without affecting the result, because  $f_0(x)$  is orthogonal to  $\Diamond(x)$  and therefore the product of the  $\Diamond(x)$  component of g(x) with  $f_0(x)$  will be equal to zero. Expressing  $g_0(x)$  as a linear combination of g(x) and  $\Diamond(x)$ , the squared modulus of  $g_0(x)$  can be rewritten as:

$$[g^{2}(x)\circ \Diamond(x)] - 1/N[g(x)\circ \Diamond(x)]^{2}.$$
(10)

Now changing the dot product to the correlation operator to apply this to a scene with unsegmented targets yields

$$\cos^{2} \theta = \frac{1}{[f_{0}(x) * f_{0}(x)]_{x=0}} \times \left\{ \frac{[g(x) * f_{0}(x)]^{2}}{[g^{2}(x) * \Diamond(x)] - 1/N[g(x) * \Diamond(x)]^{2}} \right\}.$$
(11)

This result is identical to Eq. (8) of the last section. The squared cosine ensures that the correlation plane values will be between 0 and 1.

## Signal to Noise Ratio

When the true object is present, the LACIF filter of Eq. (8) was designed to yield a value of *C* equal to one. But what is the value of *C* when there is only noise? This may be found for stationary white noise by observing that the numerator of Eq. (8) is the output of a linear filtering of the noise. For the output of a linear filter with impulse response h(x), the expected value of the squared output is equal to<sup>19</sup>

$$E[y^2(x)] = \sigma^2 E, \qquad (12)$$

where

$$E = \int_{-\infty}^{\infty} |h(x)|^2 \mathrm{d}x = R_{f_0 f_0}(0)$$
 (13)

and  $\sigma^2$  is the variance of the noise. So for areas where there is only random noise, the numerator of Eq. (8) is the output squared of the linear filter  $f_0$  that yields the output  $\sigma^2 E$  and the denominator is the variance of the noise multiplied by the normalizing factor, which yields

$$C(0) = \frac{\sigma^2 R_{f_0 f_0}}{N R_{f_0 f_0} \sigma^2} = \frac{1}{N},$$
(14)

where N is the number of pixels in the reference target. Note that this value is independent of the noise variance, so that for disjoint random noise, the signal to noise ratio is independent of the target intensity and of the noise level! When the additive noise also degrades the target af(x), the numerator of Eq. (8) becomes

$$[(af+n)*f_0]^2 = (aR_{f_0f_0} + R_{f_0})^2.$$
(15)

If the number of pixels is large, the cross correlation  $R_{fn}$  is much smaller than the autocorrelation and can be neglected in the ratio of Eq. (8). The denominator contains the variance of the signal plus the noise, and we use the fact that the sum of variances of uncorrelated processes is the sum of their variances:

$$\sigma_{af+n}^2 = a^2 \sigma_f^2 + \sigma_n^2,$$

and Eq. (8) becomes

$$C \simeq \frac{a^2 R_{f_0 f_0}^2}{N R_{f_0 f_0} (a^2 \sigma_{f_0}^2 + \sigma_n^2)} = \frac{N^2 a^2 \sigma_{f_0}^4}{N^2 \sigma_{f_0}^2 (a^2 \sigma_{f_0}^2 + \sigma_n^2)}, \quad (16)$$

$$C \simeq \frac{a^2 \sigma_f^2}{a^2 \sigma_f^2 + \sigma_n^2}.$$
(17)

It is easy to see from this expression that in the absence of noise, as we already shown,

 $\sigma_n^2\,\rightarrow\,0$ 

vields

$$C \rightarrow 1,$$
 (18)

and when

$$\frac{\sigma_n^2}{a^2 \sigma_f^2} \gg 1,$$

$$C \cong \frac{a^2 \sigma_f^2}{\sigma_n^2}.$$
(19)

So for a given true target, the correlation peak decreases approximately with the inverse of the noise, and for a given noise level, the correlation peak is proportional to the intensity. This means that a high level of random noise degrades the intensity invariance of the method. We now show that for high levels of noise, the method sacrifices intensity invariance to attain a signal to noise ratio identical to that of the matched filter.

For the matched filter, the normalized correlation peak in the presence of signal plus noise has a value

$$\begin{split} C_m &= \frac{|[af(x) + n(x)] * f_0(x)|^2}{R_{f_0 f_0}^2} \\ &= \frac{(aR_{f_0 f_0} + R_{f_0 n})^2}{R_{f_0 f_0}^2} \\ &\approx a^2. \end{split} \tag{20}$$

When there is only noise, the value is

$$C_m = \frac{(n * f)^2}{R_{f_0 f_0}^2} = \frac{R_{f_0 n}^2}{R_{f_0 f_0}^2},$$
(21)

so the signal to noise ratio (SNR) for the matched filter is equal to

$$\mathrm{SNR}_{m} = \frac{a^{2}R_{ff}}{\sigma_{n}^{2}} = \frac{a^{2}N\sigma_{f}^{2}}{\sigma_{n}^{2}}, \qquad (22)$$

where we have dropped the zero index from  $f_0$  as well as the explicit dependence on the spatial variable x, since we are only interested in the value for x = 0.

From our previous results, the signal to noise ratio for the LACIF method is equal to

$$\mathrm{SNR} = \frac{a^2 N \sigma_f^2}{a^2 \sigma_f^2 + \sigma_n^2}.$$
 (23)

So the ratio of the signal to noise ratios is

$$R = \frac{\mathrm{SNR}}{\mathrm{SNR}_m} = \frac{\sigma_n^2}{a^2 \sigma_f^2 + \sigma_n^2} = \frac{1}{1 + \frac{a^2 \sigma_f^2}{\sigma_n^2}}.$$
 (24)

When the noise is very small,

$$\sigma_n^2 \ll a^2 \sigma_f^2$$

$$R \approx \frac{\sigma_n^2}{a^2 \sigma_f^2}$$
(25)

and the signal to noise ratio of the LACIF is inferior to that of the matched filter when it is not needed, but intensity invariance is maintained as previously shown. But when the noise increases, the ratio Rtends to one, which means that the signal to noise ratio of the LACIF tends toward that of the matched filter, at the cost of some loss of invariance. In the next section we show that performance in the presence of correlated noise is another story altogether.

## Peak to Sidelobe Ratio

In this section we consider disjoint correlated noise, for which a good measure of performance is the peakto-sidelobe ratio (PSR), which is the ratio of the value of the correlation peak to the cross-correlation values in the correlation plane. In general, for lowintensity targets and high clutter, the cross correlations can be higher than the autocorrelations.

For the LACIF we have shown that in the presence of a true target, the correlation value is C = 1 independently of the intensity of the target. For clutter g(x), the LACIF yields

$$C_{g} = \frac{(g * f_{0})^{2}}{NR_{ff}\sigma_{g}^{2}} = \frac{R_{f_{0}g}^{2}}{NR_{ff}\sigma_{g}^{2}} = \frac{R_{f_{0}g}^{2}}{R_{ff}R_{gg}}.$$
 (26)

The PSR is equal to

$$\text{PSR}_f = \frac{C}{C_g} = \frac{R_{ff}R_{gg}}{R_{fg}^2},$$
(27)

where  $R_{fg}$  is the correlation between the true target f(x) and the clutter g(x). Note that this value is independent of the target intensity, and it is easy to show from the Schwartz inequality that the PSR is always greater than one. This means that no correlation peaks in the clutter can be higher than that of a true target, even if the latter has a very low intensity.

For the matched filter  $f_0(x)$ , we have shown that the normalized correlation peak value for a true target af(x) + b is  $C_{mf} = a^2$ . For clutter g(x), the normalized correlation is equal to

$$C_{mf} = \frac{(g * f_0)^2}{R_{ff}^2} = \frac{R_{fg}^2}{R_{ff}^2},$$
(28)

and the PSR is equal to

$$PSR_{m} = \frac{a^{2}R_{ff}^{2}}{R_{fg}^{2}}.$$
 (29)

The ratio of the PSR for the LACIF to that of the matched filter is equal to

$$Q = \frac{\text{PSR}}{\text{PSR}_m} = \frac{R_{gg}}{a^2 R_{ff}}.$$
(30)

The worst case of clutter is false targets, in which case  $R_{gg} \approx R_{ff}$ , which yields

$$Q \approx 1/a^2. \tag{31}$$

For low-intensity targets in clutter,  $a^2 \ll 1$ , in which case Eq. (30) shows that the performance of the matched filter is strongly degraded with respect to that of the LACIF.

In sum, compared to the matched filter, the LACIF maintains intensity invariance in the presence of weak random noise and has a signal to noise ratio performance that tends to that of the zero-mean matched filter as the random noise increases. For correlated noise the LACIF maintains an excellent PSR that is independent of the target intensity. The



Fig. 1. Target images: true and false targets.

following experiments confirm those theoretical results, as well as the superiority of the LACIF to other filters for which we have not carried out the theoretical calculations, because it would require too much space.

## Results

To demonstrate the LACIF we have chosen two slightly different vehicle images shown in Fig. 1, which have gray-level values from 0 to 255, corresponding to the output of an 8-bit camera (but note that calculations were carried out in floating point). We pasted 16 replicas of each tank on a background image that represents a natural environment (Fig. 2). Each replica has undergone an intensity transformation given by Table 1, where a and b are the parameters of Eq. (1). In the bottom part of the image is a set of replicas of the other false target vehicle with the same corresponding intensity modifications. The true targets are the ones in the upper part of the image. The numbers beside each target were not present during the identification process. Figure 3 shows the results of the first experiment using Eq. (8)for this scene. The results are obtained by calculating the three correlations and then computing the ratio as indicated in the expression. All 16 correlation peaks are equal to 1 and are very sharp. The rest of the correlation plane is lower than 0.3, which demonstrates good discrimination capability against correlated noise. The good performance can be un-



Fig. 2. Scene containing true (top) and false (bottom) targets. Objects in corresponding positions in the top and bottom halves have similar intensities.

Table 1. Intensity Transformation Parameters<sup>a</sup>

Target Number	Multiplicative Factor (a)	Additive Term (b)
1 and 17	0.33	-25
2 and 18	0.33	0
3 and 19	0.33	+25
4 and 20	0.50	-25
5 and 21	0.50	0
6 and 22	0.50	+25
7 and 23	0.75	-50
8 and 24	0.75	-25
9 and 25	0.75	+25
10 and 26	0.75	+50
11 and 27	1.00	-50
12 and 28	1.00	0
13 and 29	1.00	+50
14 and 30	1.50	-25
15 and 31	1.50	0
16 and 32	1.50	+25

<sup>*a*</sup>For each object.

derstood from the vector space interpretation: All images are intensity images and therefore every pixel has a positive value. In a multidimensional space this means that all vectors are in the first quadrant. Therefore the angles between images are fairly small and the squared cosine values are all close to one. But when the vectors are projected onto the subspace orthogonal to  $\Diamond(x)$  those vectors will be spread out in the vector space thus increasing the angles between them.

To determine the sensitivity of the LACIF to other uncontrolled target variations, we also carried out a simulation with the same scene using additive Gaussian noise with a standard deviation of 27 as shown in Fig. 4. Figure 5 shows that a threshold at 0.35 on the correlation plane still completely separates the true targets from the false targets and from the clutter. The same noisy image was used to compare the performance of the LACIF with that of other methods, that of Dickey *et al.*,<sup>1</sup> the phase only filter,<sup>20</sup> the homomorphic cameo filter,<sup>13</sup> the zero-mean classical matched filter,<sup>21</sup> and the pure phase correlation.<sup>15</sup> All of those methods have some degree of intensity invariance. In Table 2 we list the number of missed targets, the number of false alarms, and the total number of errors for each method. Only the LACIF yielded perfect performance, all others showing missed targets. Most missed targets correspond to darker objects, namely targets 1 through 6, for whom both the signal level and the signal to noise ratio are lower. As predicted, these results show that the matched filter in particular is not very effective against correlated noise under changes of target illumination, and this is also true of the other methods.

One might be impressed in spite of the fact that the clutter statistics are not used in the filter design, the correlation yields very low values in the correlation plane outside of the true correlation peaks, which are very sharp. In fact, we have shown above that the correlation values do not depend at all on the statis-



Fig. 3. Correlation results with only disjoint correlated noise: All of the true correlation peaks are equal.

tics of the noise, and that the peak-to-sidelobe ratio against clutter is always greater than one.

## Conclusion

We have designed a recognition method that is invariant under linear-intensity transformations and that uses three correlation operations involving local statistics. It can be applied directly to scenes containing unsegmented targets. The vector space interpretation gives insights that could be useful to the



Fig. 4. Scene with additive Gaussian noise and disjoint correlated noise.

development of new algorithms invariant to more general intensity transformations.

The LACIF is not only invariant to the intensity transformation that we considered, but it also yields very good discrimination against false targets and against correlated and random noise.

Discrimination is also very good for dark objects, which was a problem with other methods. When high random noise or low contrast is enough to threaten the performance of the LACIF, it sacrifices intensity invariance to attain a signal to noise ratio performance identical to that of the matched filter, a filter that yields the maximum signal to noise ratio against random Gaussian noise. When such performance is not needed, the LACIF maintains invariant correlation peaks against changes of intensity.

Pattern classification methods often use training sets consisting of subsets of true targets and of false targets. We have not used this approach here because there is only one true target and one false target, and our purpose was to determine how well the method can compensate for specific intensity variations under conditions where the correlated and random noise statistics are unknown. The results show that the method is very robust while maintaining a high degree of generalization under random variations of the true target. Extension of the method to cases of target classification with multiple true and false targets or the determination of error probabilities when the noise statistics are known are beyond the scope of this paper.

We emphasize that although the method uses the local statistics around a point to calculate the results,



Fig. 5. Correlation results for additive Gaussian noise.

Table 2. Comparative Performance in the Presence of Noise

Methods	Missed Target	False Alarm	Total Error
LACIF	0	0	0
Ref. 1	10	0	10
$POF^a$	10	2	12
$\mathrm{HCF}^{a}$	10	0	10
$\mathrm{CMF}^a$	10	1	11
$PPC^a$	8	0	8

<sup>a</sup>POF, phase only filter; HCF, homomorphic cameo filter; CMF, classical matched filter; PPC, pure phase correlation.

no segmentation is involved, and no *a priori* information about the values of the unknown parameters a and b is used. The whole process consists only of calculating three correlations and their ratios.

Finally we note that this method will not work when the training images are binary targets, because in such cases, the support  $\Diamond(x)$  of f(x) is equal to f(x).

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