Filter multiplexing by use of spatial code division multiple access approach

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The increasing popularity of optical communication has also brought a demand for a broader bandwidth. The trend, naturally, was to implement methods from traditional electronic communication. One of the most effective traditional methods is Code Division Multiple Access. In this research, we suggest the use of this approach for spatial coding applied to images. The approach is to multiplex several filters into one plane while keeping their mutual orthogonality. It is shown that if the filters are limited by their bandwidth, the output of all the filters can be sampled in the original image resolution and fully recovered through an all-optical setup. The theoretical analysis of such a setup is verified in an experimental demonstration. © 2003 Optical Society of America

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1. Introduction

Code Division Multiplexing Access (CDMA) technology focuses primarily on the direct sequence method of a spread spectrum.¹ Direct sequence is a spread spectrum technology in which the bandwidth of a signal is enlarged by artificially increasing the bit data rate. Breaking each bit into a number of subbits called chips does this.

The signal is divided into smaller bits by multiplying it by a pseudo-noise (PN) code. A simple multiplication of the original modulated signal by this high data rate PN code yields the division of the signal into smaller bits (which increases its bandwidth). Increasing the number of chips expands the bandwidth proportionally.

Let us now briefly describe the basic operation of the transmitter/receiver for the spread spectrum technique. We assume that there are two transmitters with two different messages to be transmitted. The messages $M_1(t)$ and $M_2(t)$ first are modulated. The output for each of the modulators is denoted by $S_1(t)$ and $S_2(t)$. After the modulator, each signal is

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multiplied by its own unique PN code, $C_1(t)$ and $C_2(t)$, and both are transmitted. Because various signals might be simultaneously transmitted from different transmitters, we represent these transmissions by simply adding their spectrums. At the receiver end, the incoming signal is the spread spectrum signal. To extract a single message, one must multiply the incoming signal by the corresponding PN code. By eliminating the PN code, we eliminate the spread spectrum effects for that particular message. The resulting signal is then passed through a bandpass filter centered at the carrier frequency. This operation selects only the desired signal while rejecting all surrounding frequencies due to other messages in the spread spectrum.

The idea of implementing spatial CDMA coding is derived from the equivalent method in communication, yet it is not similar. In this paper, we shall examine only one usage of such a method as applied to direct-filter multiplexing.

Section 2 presents the theoretical analysis of the concept. Computer simulations and experimental results are discussed in Sections 3 and 4, respectively. Section 5 concludes the paper.

2. Theoretical Analysis

A. Optical Image Coding

For usages, such as invariant pattern recognition or wavelet transform one requires the use of several filters simultaneously.^{2,3} To multiplex all the filters into one two-dimensional filtering plane we suggest using CDMA. For instance, assuming that we wish

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to use four different filters $F_1(x, y)$, $F_2(x, y)$, $F_3(x, y)$, and $F_4(x, y)$, we generate four PN-spatial masks $M_1(x, y)$, $M_2(x, y)$, $M_3(x, y)$, and $M_4(x, y)$ and combine the masks together to one spatial filter S(x, y) in the following way:

$$S(x, y) = \sum_{n=1}^{4} F_n(x, y) M_n(x, y).$$
(1)

This filter will be placed in the Fourier plane of the 4-*f* setup. In this setup, the original image is Fourier transformed and multiplied with the filter and inverse-Fourier transformed back to the image plane. Hence the image captured by the CCD camera will be the original image after convolution with the Fourier transform of the slide S(x, y).^{4,5}

B. Optical Image Retrieval

Optical retrieval of the filtered images uses a similar 4-*f* setup. The image is Fourier transformed and multiplied by the relevant filter and by the proper PN-code mask: $F_i(x, y) M_i(x, y)$. Then it is inverse-Fourier transformed and sampled by the CCD.

Multiplying by the mask $M_i(x, y)$ cancels all of the images not belonging to the relevant filter due to the orthogonal property of the masks. The need to multiply by the original filter $F_i(x, y)$ is due to aliasing created by the under sampling of the images. This aliasing occurs when the input image is created from an image sampled by a CCD and restored by use of a digital media. This subject is elaborated in Section 3, that deals with the computer simulations. Mathematically:

$$O(x, y) = \sum_{n=1}^{4} I(x, y) F_n(x, y) M_n(x, y).$$
(2)

Whereas I is the input image in the Fourier plane, O is the output of the coding setup also in the Fourier plane. R_i is the retrieved image in the Fourier plane, which is in this case the Fourier transform of the *i*th filter used.

$$R_{i}(x, y) = O(x, y)F_{i}(x, y)M_{i}(x, y)$$

$$= \left[\sum_{n=1}^{4} I(x, y)F_{n}(x, y)M_{n}(x, y)\right]$$

$$\times F_{i}(x, y)M_{i}(x, y)$$

$$= \sum_{n=1}^{4} I(x, y)F_{n}(x, y)M_{n}(x, y)$$

$$\times F_{i}(x, y)M_{i}(x, y).$$
(3)

Owing to the orthonormality of the PN codes one obtains

$$R_i(x, y) = I(x, y) F_i^2(x, y) M_i^2(x, y).$$
(4)

Now the retrieved image is sampled in the output plane, and the desired output is the zero order (i.e., the central image)

$$R_i(x, y) = I(x, y).$$
(5)

Note that the similarity to the spread spectrum approach is in the fact that a multiplication in the Fourier domain is equivalent to correlation in the image plane. The usage of the relevant filter is similar to the usage of the bandpass filter.

C. Computational Image Retrieval

An optional image-retrieval approach is presented below. This method involves simple computations applied after the encoding of the images, which is done in a identical way to the one described above. This is valid under certain assumptions.

Let us suppose that the Fourier coefficients of each of the CDMA masks contain a definite set of frequencies, which are multiples of twice the image-highest frequency. This is obvious, since a mask with more than two chips per pixel must be composed of such multiples. For the first filter these coefficients will be marked as αi , where as $i = 0, 1, 2, \ldots$, for the second filter β_i , and so on. We assume that the signals are one dimensional, for the simplicity of the explanation, although the same applies for twodimensional coding. Then on the CCD we receive several areas the size of the original image (due to convolution of the image with the different orders of the filter in the image plane), and the central one is referred to as order zero, the ones adjacent to zero order, which are identical due to symmetry and realness of input signal and filter, and is referred to as first order, and so on.

Then it is obvious that the different orders are linear combinations of the four filtered images:

order_0(n) =
$$\alpha_0 a(n) + \beta_0 b(n) + \chi_0 c(n) + \delta_0 d(n)$$
,
order_1(n) = $\alpha_1 a(n) + \beta_1 b(n) + \chi_1 c(n) + \delta_1 d(n)$,
order_2(n) = $\alpha_2 a(n) + \beta_2 b(n) + \chi_2 c(n) + \delta_2 d(n)$,
order_3(n) = $\alpha_3 a(n) + \beta_3 b(n) + \chi_3 c(n) + \delta_3 d(n)$.
(6)

Where *a*, *b*, *c*, and *d* are the separate output of the four filters accordingly, and α_i , β_i , χ_i , and δ_i are defined by the CDMA coding mask.

Let us formulate this problem in matrix form:

$$\begin{bmatrix} \alpha_{0} & \beta_{0} & \chi_{0} & \delta_{0} \\ \alpha_{1} & \beta_{1} & \chi_{1} & \delta_{1} \\ \alpha_{2} & \beta_{2} & \chi_{2} & \delta_{2} \\ \alpha_{3} & \beta_{3} & \chi_{3} & \delta_{3} \end{bmatrix} \begin{pmatrix} a_{0} & \cdots & a_{N} \\ b_{0} & \cdots & b_{N} \\ c_{0} & \cdots & c_{N} \\ d_{0} & \cdots & d_{N} \end{pmatrix} = \begin{pmatrix} \text{order0}_{0} & \cdots & \text{order0}_{N} \\ \text{order1}_{0} & \cdots & \text{order1}_{N} \\ \text{order2}_{0} & \cdots & \text{order3}_{N} \end{pmatrix}.$$

$$(7)$$

A is the matrix of the filters coefficients. A's inverse matrix (i.e. A^{-1}) can be calculated, thus enabling retrieval of the original filtered images.

$$A^{-1} \cdot \begin{pmatrix} \operatorname{order} 0_0 & \cdots & \operatorname{order} 0_N \\ \operatorname{order} 1_0 & \cdots & \operatorname{order} 1_N \\ \operatorname{order} 2_0 & \cdots & \operatorname{order} 2_N \\ \operatorname{order} 3_0 & \cdots & \operatorname{order} 3_N \end{pmatrix} = \begin{pmatrix} a_0 & \cdots & a_N \\ b_0 & \cdots & b_N \\ c_0 & \cdots & c_N \\ d_0 & \cdots & d_N \end{pmatrix}.$$
(8)

If the filter's Fourier coefficients are not ideal, e.g., as shown in Fig. 1, additional calculation is required to retrieve the desired output. Let us suppose that around each one of the filter's main frequencies (α_i , β_i , χ_i , or δ_i according to the filter) are several other coefficients, i.e., the filters are not ideal gratings. This is demonstrated in Fig. 1.

Each order can be displayed in the following way:

$$\begin{aligned} \text{order}_{i}(n) &= \alpha_{i}[a(n) + \alpha_{i,-1}a(n-1) + \alpha_{i,1}a(n \\ &+ 1)\alpha_{i,2}a(n+2)] + \beta_{i}[b(n) + \beta_{i,-1}b \\ &\times (n-1) + \beta_{i,1}b(n+1) + \beta_{i,2}b(n+2)] \\ &+ \chi_{i}[c(n) + \chi_{i,-1}c(n-1) + \chi_{i,1}c(n+1) \\ &+ \chi_{i,2}c(n+2)] + \delta_{i}[d(n) + \delta_{i,-1}d(n \\ &- 1) + \delta_{i,1}d(n+1) + \delta_{i,2}d(n+2)]. \end{aligned}$$

In the special case that $\alpha_{i,j} = \beta_{i,j} = \chi_{i,j} = \delta_{i,j}$ for $j = \ldots -2, -1, 1, 2, \ldots$ and $i = 1, 2, \ldots$, Eq. (9) can be written as

$$\begin{aligned} \text{order}_{i}(n) &= \alpha_{i}a(n) + \beta_{i}b(n) + \chi_{i}c(n) + \delta_{i}d(n) \\ &+ \alpha_{i,-1}(\alpha_{i}a(n-1) + \beta_{i}b(n-1) + \chi_{i}c \\ &\times (n-1) + \delta_{i}d(n-1)) + \alpha_{i,1}(\alpha_{i}a(n+1) + \beta_{i}b(n+1) + \chi_{i}c(n+1) + \delta_{i}d(n+1)) \\ &+ 1) + \beta_{i}b(n+1) + \chi_{i}c(n+1) + \delta_{i}d(n+2) \\ &+ 1)) + \alpha_{i,2}(\alpha_{i}a(n+2) + \beta_{i}b(n+2) \\ &+ \chi_{i}c(n+2) + \delta_{i}d(n+2)) \end{aligned}$$

$$= \text{orig_order_}_{i}(n) + \alpha_{i,-1} \times \text{orig_order_}_{i}(n+1) \\ &+ \alpha_{i,2} \times \text{orig_order_}_{i}(n+2), \quad (10) \end{aligned}$$

where orig_order_*i* is a vector describing the values of order *i* if the filter coefficients were composed only of the coefficients α_i , β_i , χ_i , and δ_i , i.e., ideal gratings:

orig_order_
$$i(n) = \alpha_i a(n) + \beta_i b(n) + \chi_i c(n) + \delta_i d(n).$$
 (11)

This is valid for CDMA masks chosen in a specific form, where the chips of each pixel are symmetric. In matrix form, order i can be formulated as

$$\begin{bmatrix} \ddots & \ddots & 0 & 0 & 0 \\ \ddots & \alpha_i & \alpha_{i,-1} & 0 & 0 \\ 0 & \alpha_{i,2} & \alpha_{i,1} & \alpha_i & \ddots \\ 0 & 0 & \ddots & \ddots & \ddots \end{bmatrix} \begin{pmatrix} \operatorname{orig_order_}i_0 \\ \vdots \\ \operatorname{orig_order_}i_N \end{pmatrix}$$
$$= B \cdot \begin{pmatrix} \operatorname{orig_order_}i_0 \\ \vdots \\ \operatorname{orig_order_}i_N \end{pmatrix} = \begin{pmatrix} \operatorname{order_}i_0 \\ \vdots \\ \operatorname{order_}i_N \end{pmatrix}. \quad (12)$$

Whereas *B* is a matrix, which is composed of a main diagonal, which has a constant value of α_i , the diagonal above the main has the constant value of $\alpha_{i,-1}$. The diagonal below the main has the constant value of $\alpha_{i,1}$, and the one below that has main has the constant value of $\alpha_{i,2}$. The rest of the matrix is zeros. orig_order_*i* can easily be retrieved by use of the following formula:

$$B^{-1} \cdot \begin{pmatrix} \operatorname{order}_{i_0} \\ \vdots \\ \operatorname{order}_{i_N} \end{pmatrix} = \begin{pmatrix} \operatorname{orig_order}_{i_0} \\ \vdots \\ \operatorname{orig_order}_{i_N} \end{pmatrix}.$$
(13)

Now orig_order_*i* can be inserted into Eq. (8) with the values α_i , β_i , χ_i , and δ_i , and the output of the filters can be calculated.

This algorithm can be used to retrieve the output of the filters used, with a relatively small computational complexity. The values of A, B, and A^{-1} , B^{-1} are calculated only once beforehand, and they are independent of the input image. Multiplying A^{-1} by the $N \times 4$ matrix is O(N) (whereas N is the number of pixels in each order), and multiplying B^{-1} with a vector of length N is of O(N) because B^{-1} is a very sparse matrix. (B is a matrix with a small amount of coefficients around its diagonal.) Therefore the retrieval of the output is O(N). For two-dimensional images, the algorithm is identical and the complexity is $O(N^2)$ accordingly.

Calculating the outputs of the filters only by computer is of complexity $O(N \log N)$, whereas by this method, the complexity is O(N). This advantage is also substantial in two-dimensional images, where the complexity of a fully digital computation is $O(N^2 \log N)$, whereas here it is simply $O(N^2)$. Note also that storing information in CDMA-coded form is more redundant to noise.

Notice that while sampling the image, the number of pixels sampled remained the same as the original image; therefore there was no need to increase the number of pixels sampled.

3. Computer Simulations

A. General

A MATLAB simulation was constructed. An input image of 64 by 64 pixels was chosen. The filter was composed out of three subfilters, and therefore required at least three chips per pixel. The PN-spatial



Fig. 1. One of the filter's discrete spectrums.

masks were composed of vertical lines for the sake of simplicity. Furthermore, to keep the main coefficients of the different filters' spectrums coinciding, the chips were made to be symmetrical in each pixel. Therefore, six chips were chosen for each pixel. The masks are represented by $M_1(x, y)$, $M_2(x, y)$, and $M_3(x, y)$, and they are orthogonal and binary. This condition suffices for CDMA trivial encoding and decoding. The filters are denoted as $F_1(x, y)$, $F_2(x, y)$, and $F_3(x, y)$. Note that the masks and the filters must have symmetrical properties to create a real image on the CCD, which captures only intensities. In case the input image or the filtering functions causes a non-real output, the phase information may be recovered by use of digital holography.⁶

To sample the created image in its original resolution, we limit the filters to have only a quarter of the original bandwidth in each axis. The chosen filter may be seen in Fig. 2. When sampling the output image, one must sample in the full resolution in the horizontal plane to fully retrieve the coding masks, which are composed of vertical grids. On the horizontal plane, it suffices to sample in one quarter of the original resolution. Hence the image is sampled in a resolution of 256 by 16 pixels, which is equivalent to the number of pixels in the original image (64 by 64).

The image itself is sampled and reconstructed computationally. The image is inverse-Fourier transformed, multiplied by the matched mask $M_i(x, y)$, and filtered. One may see a complete retrieval of the spectrum of the original image. Note that the similarity to the spread-spectrum approach is in the fact that a multiplication in the Fourier domain is equivalent to correlation in the image plane. The usage of the filter is similar to the usage of the bandpass filter.

Furthermore, the filtered images are retrieved by use of the calculation proposed in Subsection 2.C, and







by applying a simple program with a complexity of $O(N^2)$ (this is a two-dimensional image).

B. Simulation Results

For the simulations, a spatial chirp image having 64 by 64 pixels was chosen. The chosen filters are seen in Fig. 2. Note that a deliberate overlapping was



Fig. 4. Ideal output of the different filters.

created between the filters to demonstrate the generality of the suggested approach. Obviously, this is not obligatory.

The output of each one of the four filters was computed separately. Then the output of the filters was calculated after multiplexing according to the technique described in Subsection 2.C, and by using the





Fig. 5. Retrieved output from the experiment.

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under sampled image. This produced satisfactory results for the direct output reconstruction and the computational technique.

The fact that the computational image retrieval produced almost identical results to the direct (and computationally costly) retrieval shows that by a proper choice of the coding masks, there is no degradation in performance by usage of this method, yet computationally many iterations are spared. Now this method needed to be proved experimentally.

4. Experimental Results

To verify the simulation the method was tested in a 4-*f* setup. The input was identical to the input in the simulation and shown in Fig. 3.

The filter mask was identical to the one used in the simulation. The ideal output of each filter was calculated just by using the input mask, with no multiplexing and no undersampling is shown in Fig. 4.

The retrieved output, as calculated with the computational technique presented in Subsection 2.C from the undersampled experimental output is shown in Fig. 5.

As can be seen, the different outputs are similar to the ones calculated ideally. This proves this method worked experimentally as well. The fact that the masks and filters were not ideal, having finite dimensions, and were sampled by a CCD had no visible degrading effect.

Notice that here binary-coding masks and filters were used. This simplifies the production of the masks but is not obligatory.

5. Conclusions

From the experimental results and the simulation results one can see that CDMA multiplexing offers varied implementations with a low computational This method can be very effective for invariant cost. pattern recognition and wavelet transformation, where there is a need to use several filters on the input simultaneously. Furthermore, because each multiplexed filter bandwidth is limited, there is no need to multiplex the image in higher resolution.

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