POSTGRAU MÁSTER EN CIENCIAS ACTUARIALES Y FINANCIERAS

Ejercicios 6B (continua) resueltos

13.-Si la función de cohorte es **l(x)=(100-x)/**190 , y la tasa instantánea de mortalidad es

$\mu_{40} = \frac{1}{100-x} = 0{,}01667$ $\mu_{40} = 0{,}01667$ para, evidentemente, una persona de 40 años. Calcular la esperanza de vida para dicha persona. Utilizando métodos.

a) con T(x) y l(x) sería

$$\bar{e}_x = \bar{e}_{40} = \frac{T_{40}}{l_{40}} = \frac{9{,}47368}{0{,}315789} = 29{,}99999 \cong 30$$

$$T_{40} = \int_{40}^{100} (100-x)/190dx = \frac{1}{190}\left[100x - \frac{x^2}{2}\right]_{40}^{100} = \frac{1}{190}(5000-3200)$$
$$= 9{,}47368$$

$$l_{40} = \frac{100-40}{190} =$$

b) con

$$\bar{e}_x = \bar{e}_{40} = \int_0^{w-x} t \cdot \ {}_tp_x \cdot \mu(x+t)dt = \int_0^{60} t \cdot \frac{(60-t)}{60} \cdot \frac{1}{(60-t)} dt = \frac{1}{60}\int_0^{60} t\, dt$$
$$= \frac{1}{60}\left[\frac{t^2}{2}\right]_0^{60} = 30$$

c) con

$${}_tp_x = \ {}_tp_{40} = \frac{l_{40+t}}{l_{40}} \frac{\frac{100-40-t}{190}}{\frac{100-40}{190}} = \frac{60-t}{60}$$

$$= \bar{e}_{40} = \int_0^{w-x} \cdot \ {}_tp_x \cdot dx = \int_0^{60} \frac{60-t}{60} \cdot dx = \frac{1}{60}\int_0^{60} 60 - t \cdot dx = \frac{1}{60}\left[60t - \frac{t^2}{2}\right]_0^{60}$$
$$= \frac{1}{60}[3600-1800] = 30$$

14.-Sabiendo que l(x)=100-x (para x≤100) obtener la esperanza de vida de una persona de 50 años.

$$\bar{e}_x = \bar{e}_{50} = \int_0^{100-50} \cdot\ {}_tp_x dt = \int_0^{100-50} 1 - \frac{t}{50} \cdot dt = \left[t - \frac{t^2}{50 \cdot 2}\right]_0^{50} = 25$$

$${}_tp_x = {}_tp_{50} = \frac{l_{50+t}}{l_{50}} == \frac{100 - 50 - t}{100 - 50} = 1 - \frac{t}{50}$$

también:

$$\mu_x = -\frac{\grave{l}(x)}{l(x)} = -\frac{-1}{100 - x} = \frac{1}{100 - x} \Rightarrow \mu_{50} = \frac{1}{50}$$

$$\mu_{50+t} = \frac{1}{100 - (50 + t)} = \frac{1}{50 - t}$$

$$\bar{e}_x = \bar{e}_{50} = \int_0^{w-x} t \cdot\ {}_tp_x \cdot \mu(x + t)dt = \int_0^{w-x} t \cdot \left(1 - \frac{t}{50}\right) \cdot \frac{1}{50 - t} dt$$
$$= \frac{1}{50}\int_0^{w-x} t \cdot (50 - t) \cdot \frac{1}{50 - t} dt = \frac{1}{50}\left[\frac{t^2}{2}\right]_0^{50} = 25$$

15.- Con la función de cohorte **l(x)=(100-x)/**190 obtener la esperanza de vida conjunta de una pareja de 50 y 60 años.

$\bar{e}_{xy} = \bar{e}_{50:60} = \int_0^{100-\max(x,y)} t \cdot\ {}_tp_{xy} \cdot \mu(x + t, x + t)dt = \int_0^{40} t \cdot$
${}_tp_x \cdot\ {}_tp_x \cdot \mu(x + t, x + t)dt = \int_0^{w-x} t \cdot \left(\frac{40-t}{40}\right) \cdot \left(\frac{50-t}{50}\right) \cdot (\frac{40-t+50-t}{(40-t)(50-t)})dt =$
$\frac{1}{40\cdot50}\int_0^{w-x} t \cdot (40 - t + 50 - t)dt = \frac{1}{40\cdot50}\int_0^{w-x} 40t - t^2 + 50t - t^2 dt =$
$\frac{1}{40\cdot50}\int_0^{w-x} 90t - 2t^2 dt = \frac{1}{2000}\left[\frac{90t^2}{2} - \frac{2t^3}{3}\right]_0^{40} = \frac{1}{2000}[72000 - 42666{,}6]=$**14,6666**

$${}_tp_x = {}_tp_{60} = \frac{l_{60+t}}{l_{60}} \frac{\frac{100 - 60 - t}{190}}{\frac{100 - 60}{190}} = \frac{40 - t}{40}$$

$${}_tp_x = {}_tp_{50} = \frac{l_{50+t}}{l_{50}} \frac{\frac{100 - 50 - t}{190}}{\frac{100 - 50}{190}} = \frac{50 - t}{50}$$

$$\mu_x = -\frac{\grave{l}(x)}{l(x)} = -\frac{-\frac{1}{190}}{\frac{100 - x}{190}} = \frac{1}{100 - x}$$

$$\mu_{60+t} = \frac{1}{40 - t}$$

$$\mu_{50+t} = \frac{1}{50 - t}$$

16.-Con la función de cohorte **l(x)=(100-x)/**190 obtener la esperanza de vida conjunta hasta la extinción de una pareja de 50 y 60 años.

$$\bar{e}_x = \bar{e}_{50} = \frac{T_{50}}{l_{50}} = \frac{9{,}689922}{0{,}2631578} = 24{,}9999 \cong 25$$

$$T_{50} = \int_{50}^{100} (100-x)/190dx = \frac{1}{190}\left[100x - \frac{x^2}{2}\right]_{50}^{100} = \frac{1}{190}(5000-3750)$$
$$= 6{,}578947$$

$l_{50} = \frac{100-50}{190}$ =0,26315789

$\bar{e}_x = \bar{e}_{60} = \frac{T_{60}}{l_{60}} = \frac{4{,}2105263}{0{,}2105263} = 19{,}9999 \cong 20$

$$T_{60} = \int_{60}^{100} (100-x)/190dx = \frac{1}{190}\left[100x - \frac{x^2}{2}\right]_{60}^{100} = \frac{1}{190}(5000-4200)$$
$$= 4{,}2105263$$

$l_{60} = \frac{100-60}{190}$ =0,2105263

$\bar{e}_{\overline{50:60}} = \bar{e}_{50} + \bar{e}_{60}$- $\bar{e}_{50:60} = 25 + 20 - 14{,}6666 = 30{,}333$

17.- Si la función $\mu(x) = \frac{1}{100-x}\ para\ x \in [0{,}100]$

Calcular la esperanza de vida para una persona de 50 años

$$\mu(50) = \frac{1}{100-50} = \frac{1}{50}$$

$\bar{e}_{50} = \int_0^{50} t\, \mu(50)dt = \frac{1}{50}\left[\frac{t^2}{2}\right]_0^{50} = \frac{1}{50} \cdot 1250 = 25.$ en este caso

$${}_tp_x = e^{-\int_x^{x+t} \mu(y)dy} = e^{-\int_x^{x+t} \frac{1}{100-y}dy} = \frac{100-x-t}{100-x}$$

$$\bar{e}_{50} = \int_0^{50} \frac{50-t}{50}dt = \frac{1}{50}\left[50t - \frac{t^2}{2}\right]_0^{50} = \frac{1}{50}[1250] = 25$$

$$\bar{e}_{50} = \int_0^{50} t\left(\frac{100-x-t}{100-x}\right)\left(\frac{1}{100-x-t}\right)dt = \int_0^{50} t\left(\frac{1}{100-50}\right)dt =$$

18.- Calcular con $\mu(x)=\frac{1}{100-x}\ para\ x\in[0,100]$ la probabilidad de que una pareja de la misma cohorte de 30 y 40 años el mayor fallezca antes de 10 años mientras que el otro sobrevive

conocemos que

$$_tp_x=e^{-\int_x^{x+t}\mu(y)dy}=e^{-\int_x^{x+t}\frac{1}{100-y}dy}=\frac{100-x-t}{100-x}$$

$$_tp_x=e^{-\int_x^{x+t}\mu(y)dy}=e^{-\int_x^{x+t}\frac{1}{100-y}dy}=\frac{100-x-t}{100-x}$$

$$_{10}q_{40}\cdot{}_{10}p_{30}=\left(1-\frac{100-40-10}{100-40}\right)\cdot\left(\frac{100-30-10}{100-30}\right)=(1-\frac{50}{60})\cdot\left(\frac{60}{70}\right)=\frac{600}{4200}$$
$$=0{,}0142857$$

19. Con la información $_tp_x=\frac{100-x-t}{100-x}$

hallar la esperanza de vida hasta la disolución de una pareja , cuyas edades son de 30 y 40 años

$$\bar{e}_{xy}=\bar{e}_{30:40}=\int_0^{100-\max(x,y)}t\cdot\ {}_tp_{xy}\cdot\mu(x+t,y+t)dt$$
$$=\int_0^{60}t\cdot\ {}_tp_x\cdot\ {}_tp_y\cdot\mu(x+t,y+t)dt$$
$$=\int_0^{w-x}t\cdot\left(\frac{70-t}{70}\right)\cdot\left(\frac{60-t}{60}\right)\cdot\left(\left(\frac{1}{70-t}\right)+\left(\frac{1}{60-t}\right)\right)dt$$
$$=\int_0^{w-x}t\cdot\left(\frac{70-t}{70}\right)\cdot\left(\frac{60-t}{60}\right)\cdot\left(\frac{60-t+70-t}{(70-t)(60-t)}\right)dt$$
$$=\frac{1}{70\cdot60}\int_0^{w-x}t\cdot(70-t)(60-t)\left(\frac{60-t+70-t}{(70-t)(60-t)}\right)dt=$$

$$\frac{1}{70\cdot60}\int_0^{w-x}t(60-t+70-t)dt=\frac{1}{70\cdot60}\int_0^{w-x}60t-t^2+70t-t^2dt=$$
$$\frac{1}{70\cdot60}\int_0^{w-x}130t-2t^2dt=\frac{1}{4200}\left[\frac{130t^2}{2}-\frac{2t^3}{3}\right]_0^{60}=\frac{234000-144000}{4200}=21{,}428$$

20.- Con la información $_tp_x=\frac{100-x-t}{100-x}$

hallar la esperanza abreviada de vida hasta la disolución de una pareja , cuyas edades son de 30 y 40 años

$$ {}_{t}p_{30} = \frac{100 - x - t}{100 - x} = \frac{100 - 30 - t}{100 - 30} = \frac{70 - t}{70} $$

$$ {}_{t}p_{40} = \frac{100 - x - t}{100 - x} = \frac{100 - 40 - t}{100 - 40} = \frac{60 - t}{60} $$

$$ e_{30:40} = \frac{1}{70 \cdot 60} \sum_{t=1}^{59} (70 - t)(60 - t) = 20{,}928 $$

$$ e_{30} = \frac{1}{70} \sum_{t=1}^{69} (70 - t) = 34{,}5 $$

$$ e_{40} = \frac{1}{60} \sum_{t=1}^{59} (60 - t) = 29{,}5 $$

21.- En el anterior esperanza de vida abreviada hasta la extinción

$$ e_{\overline{30:40}} = \frac{1}{70} \sum_{t=1}^{69} (70 - t) + \frac{1}{60} \sum_{t=1}^{59} (60 - t) - \frac{1}{70 \cdot 60} \sum_{t=1}^{69} (70 - t)(60 - t) = 43{,}07 $$