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Resúmenes

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Homenaje a
Richard Aron
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Regularization by sup-inf convolutions on Riemannian manifolds: an extension of Lasry-Lions theorem to manifolds of bounded curvature

Daniel Azagra *Universidad Complutense de Madrid*

We show how Lasry-Lions's result on regularization of functions defined on $\mathbb{R}^n$ or on Hilbert spaces by sup-inf convolutions with squares of distances can be extended to (finite or infinite dimensional) Riemannian manifolds M of bounded sectional curvature. More specifically, among other things we show that if the sectional curvature K of M satisfies $-K_0 \leq K \leq K_0$ on M for some $K_0 > 0$, and if the injectivity and convexity radii of M are strictly positive, then every bounded, uniformly continuous function $f : M \to \mathbb{R}$ can be uniformly approximated by globally $C^{1,1}$ functions defined by

$$(f_\lambda)^\mu = \sup_{z\in M} \inf_{y\in M} \{f(y) + \frac{1}{2\lambda} d(z,y)^2 - \frac{1}{2\mu} d(x,z)^2\}$$

as $\lambda, \mu \to 0^+$, with $0 < \mu < \lambda/2$. Our definition of (global) $C^{1,1}$ smoothness is intrinsic and natural, and it reduces to the usual one in flat spaces but, in the noncompact case, this definition differs from other notions of (rather local) $C^{1,1}$ smoothness that have been used by other authors (based on charts).

The importance of this regularization method lies (rather than on the degree of smoothness obtained) on the fact that the correspondence $f \mapsto (f_\lambda)^\mu$ is explicit and preserves many significant geometrical properties that the given functions f may have, such as invariance by a set of isometries, infima, sets of minimizers, ordering, local or global Lipschitzness, and (only when one additionally assumes that $K \leq 0$) local or global convexity.

We also give two examples showing that this result completely fails, even for (nonflat) Cartan-Hadamard manifolds, whenever f or K are not bounded.

This is joint work with Juan Ferrera (UCM).

On convex-cyclic operators

Antonio Bonilla *Universidad de la Laguna*

A bounded linear operator T on Banach space X is called *convex-cyclic* if there exists a vector $x \in X$ such that the convex hull of $Orb(T,x) = \{T^n x : n = 0, 1, \cdots\}$ is dense in X. We give a Hahn-Banach characterization for convex-cyclicity. We also obtain examples of a bounded linear operator S with $\sigma_p(S^*) = \emptyset$ in every Banach spaces such that S is convex-cyclic, but S is not weakly hypercyclic and examples in some Banach spaces such that S is convex-cyclic and S^2 is not convex-cyclic . We also characterize the diagonalizable normal operators that are convex-cyclic and give a condition on the eigenvalues of an arbitrary operator for it to be convex-cyclic that we apply adjoint multiplication operators.

Joint work with Teresa Bermúdez and N. S. Feldman

Multilinear stability of classes of vector-valued sequences

Geraldo Botelho *Universidade Federal de Uberlândia – Brasil*

In this talk we study the stability of some frequently used classes of vector-valued sequences under the transformation by continuous multilinear operators between Banach spaces. We investigate the behavior of the following Banach spaces of vector-valued sequences: absolutely summable sequences, weakly summable sequences, norm null sequences, weakly null sequences, bounded sequences, unconditionally summable sequences, almost unconditionally summable sequences, Cohen strongly summable sequences.
This is a joint work with Jamilson Campos.
G. Botelho is supported by CNPq Grant 302177/2011-6 and Fapemig Grant PPM-00326-13.

On Random Unconditional Convergence in rearrangement invariant spaces

Guillermo P. Curbera *Universidad de Sevilla*

The concept of Random Unconditional Convergence (RUC, in short) in Banach spaces was introduced by Billard, Kwapień, Pełczyński, and Samuel in the 1980s. Regarding RUC in rearrangement invariant spaces there is a remarkable result by Dodds, Semenov, and Sukochev relating RUC systems and orthonormal uniformly bounded systems. We present an extension of this last result.
Joint work with Sergey V. Astashkin (Samara State University, Russia) and Konstantin E. Tikhomirov (University of Alberta, Canada)

Existence of unpreserved extreme points in the disc algebra $\mathbb{A}$

Antonio José Guirao *Universidad Politécnica de Valencia*

An extreme point of the unit ball of a Banach space X is called *unpreserved* if it is not an extreme point of the unit ball of its bidual. As soon as this concept arose in Phelps' investigations in the 60's, he wondered whether such a points really exist. The answer was suggested by K. de Leeuw and justified by Katznelson in a private note to Phelps who wrote an *added in proof* comment in his paper: the disc algebra contains unpreserved extreme points. As far as we know, there exists no concrete reference to the argument used by Katznelson. Therefore, we present here our naive approach to this result.
This is a joint work with V. Montesinos

Nonexistence of certain universal polynomials between Banach spaces

Joaquín Gutiérrez *Universidad Politécnica de Madrid*

A well-known result due to W. B. Johnson (1971) asserts that the formal identity operator from ℓ_1 into ℓ_∞ is universal for the class of non-compact operators between Banach spaces. We show that there is neither a universal non-compact polynomial nor a universal non-unconditionally converging polynomial between Banach spaces.
Joint work with Raffaella Cilia.

Information and σ-algebras

Carlos Hervés *Universidad de Vigo*

In this work, we clarify the relationship between the information that an agent receives from a signal, from an experiment or from his own ability to determine the true state of nature that occurs and the information that an agent receives from a σ-algebra. We show that, for countably generated σ-algebras, the larger it is, the larger the information is. The same is true for general σ-algebras after the removal of a negligible set of states.

Fixed Point Property in Banach spaces and some connections with Renorming Theory

María Ángeles Japón *Universidad de Sevilla*

A Banach space is said to have the Fixed Point Property (FPP) if every nonexpansive mapping defined from a closed convex bounded subset into itself has a fixed point. Recall that a mapping $T : C \to C$ is said to be nonexpansive if

$$||Tx - Ty|| \leq ||x - y||$$

for every $x, y \in C$.
The nonexpansiveness of a mapping strongly depends on the underlying norm. In fact, there are Banach spaces which fail to have the FPP but they are still FPP-renormable. It is also known that there are some Banach spaces which do not admit an equivalent norm with the FPP. We will talk about these results and some related open problems.

Spectral theory and orthogonally additive polynomials

José Luis G. Llavona *Universidad Complutense de Madrid*

We prove several results which establish new relations between the theory of orthogonally additive polynomials and the spectral theory of self-adjoint operators.

Topologies on spaces of holomorphic functions

Jerónimo López-Salazar Codes *Universidad Politécnica de Madrid*

In this talk we study the coincidence of the τ_ω and the τ_δ topologies on the space of all holomorphic functions defined on an open subset U of a Banach space. Dineen and Mujica proved that $\tau_\omega = \tau_\delta$ when U is a balanced open subset of a separable Banach space with the bounded approximation property. In this talk, we consider the $\tau_\omega = \tau_\delta$ problem for several types of non-balanced domains U.

The Bishop-Phelps-Bollobás version of Lindenstrauss properties A and B

Miguel Martín *Universidad de Granada*

We study a Bishop-Phelps-Bollobás version of Lindenstrauss properties A and B. For domain spaces, we study Banach spaces X such that (X, Y) has the Bishop-Phelps-Bollobás property (BPBp) for every Banach space Y. We show that in this case, there exists a universal function $\eta_X(\varepsilon)$ such that for every Y, the pair (X, Y) has the BPBp with this function. This allows us to prove some necessary isometric conditions for X to have the property. We also prove that if X has this property in every equivalent norm, then X is one-dimensional. For range spaces, we study Banach spaces Y such that (X, Y) has the Bishop-Phelps-Bollobás property for every Banach space X. In this case, we show that there is a universal function $\eta_Y(\varepsilon)$ such that for every X, the pair (X, Y) has the BPBp with this function. This implies that this property of Y is strictly stronger than Lindenstrauss property B. The main tool to get these results is the study of the Bishop-Phelps-Bollobás property for c_0-, ℓ_1- and ℓ_∞-sums of Banach spaces.

Tauberian Polynomials

Luiza A. Moraes *Universidade Federal do Rio de Janeiro – Brasil*

We say that a continuous n-homogeneous polynomial $P : E \to F$ is *Tauberian* if $\tilde{P}^{-1}(F) \subset E$ or, equivalently, if $\tilde{P}(E'' \setminus E) \subset F'' \setminus F$, where $\tilde{P}$ denotes the Aron-Berner extension of P (which is a polynomial from E'' into F''). In case of a continuous linear operator $T : E \to F$, this condition means that $T^{**-1}(F) \subset E$ (where T^{**} denotes a bi-transpose of T). Continuous linear operators satisfying this condition have been studied in [3] by N. Kalton and A. Wilansky, who refered to them as *Tauberian operators*. Tauberian operators were introduced to investigate a problem in summability theory from an abstract point of view. Afterwards, they have made a deep impact on the isomorphic theory of Banach spaces. A vast literature has been devoted to them; we refer to the recent monography [2] by M. González and A. Martínez-Abejón for details.

In this talk we will see how some of the ideas behind the notion of Tauberian operator can be seen from a multilinear or polynomial point of view. When loosing linearity new difficulties arise and different tools are required. We will show to which extent results valid for Tauberian operators still hold in such non linear setting. In special we will present a number of examples that will clarity how the behavior of Tauberian polynomials differs from that of Tauberian operators.

All the results presented in this talk were obtained in collaboration with Maria D. Acosta (Universidad de Granada) and Pablo Galindo (Universidad de Valencia) and are part of [1].

References

[1] M. D. Acosta, P. Galindo y L. A. Moraes, *Tauberian polynomials*, J. Math. Anal. Appl. 409 (2014) 880-889.

[2] M. González y A. Martínez-Abejón, *Tauberian Operators*, Operator Theory: Advances and Applications, vol. **194**, Birkhäuser Verlag, Basel, 2010.

[3] N. J. Kalton y A. Wilansky, *Tauberian operators on Banach spaces*, Proc. Amer. Math. Soc. 57 (1976) 251-255.

On a characterization of continuity

Gustavo A. Muñoz-Fernández *Universidad Complutense de Madrid*

It is well known that a continuous function $f : \mathbb{R} \to \mathbb{R}$ transforms compact sets into compact sets and connected sets into connected sets. These two results are a part of any standard course on elementary real analysis in one variable. It is equally easy, but not as well known, that if a function $f : \mathbb{R} \to \mathbb{R}$ transforms compact sets into compact sets and connected sets into connected sets, then f is necessarily continuous. In this talk we will show what happens when we drop only one of the previous two conditions that guaranty the continuity of a function $f : \mathbb{R} \to \mathbb{R}$. We will also point out that when we consider the same question for polynomials on an infinite dimensional normed space, the conclusion we arrive at is completely different. Joint work with José Luis Gámez, Daniel Pellegrino and Juan B. Seoane-Sepúlveda.

The λ-function of Aron and Lohman on Jordan structures

Antonio M. Peralta *Universidad de Granada*

When R. Aron and R. Lohman introduced the λ-function on normed spaces, they also posed the challenge of determining what spaces of operators satisfy the λ-property, and to describe the exact form of the λ-function in these spaces. This defiance motivated L.G. Brown and G.K. Pedersen to write a whole series of papers exploring the λ-function on von Neumann and C*-algebras. Their studies gave rise to the notion of Brown-Pedersen quasi-invertibility in C*-algebras. As a result of these studies, the λ function at an element a in the closed unit ball of a C*-algebra A was completely determined in terms of the distances from a to the set of Brown-Pedersen quasi-invertible elements in A or to its complement. Among the consequences they proved that every von Neumann algebra satisfies the uniform λ-property. There is a class of complex Banach spaces determined by the nice holomorphic properties of their closed unit balls, the elements of this class are called JB*-triples, and every C*-algebra lies in the strictly wider class of JB*-triples, which also contains Hilbert spaces and spin factors. We shall present in this talk some new generalizations of the results established by Brown and Pedersen to the setting of JB*-triples. New estimations of the λ-function on the closed unit ball of a JB*-triple will allow us to show that every JBW*-triple (i.e. a JB*-triple which is a dual Banach space) satisfies the uniform λ-property.

Undecidability of the Spectral Gap Problem

David Pérez García *Universidad Complutense de Madrid*

When talking about different phases in physics, the first thing that comes to mind is the division in solid, liquid and gas. At zero (or close to zero) temperature, where quantum mechanics is the physical law that governs the system, there are also different phases.The exotic and unexpected properties of some of these quantum phases, like superconductivity, superfluidity, fractional statistics, topological dependency, etc. have attracted the attention of physicists for many years. Mathematically, the definition of quantum phase is given by the existence of a gap in the spectrum of the Hamiltonian that encodes the interactions of the system. This is why some of the most famous open problems in quantum many body physics ask about the existence/absence of spectral gap in concrete models. In this talk, aimed at non-specialists, I will present our recent result: the existence of spectral gap is an undecidable problem. This explains clearly why there is no known general criterion to decide the existence of spectral gap. Our result implies that there cannot be any.
Joint work with T. S. Cubitt and M.M. Wolf.

One-side James Theorem

Antonio Pérez Hernández *Universidad de Murcia*

James' Compactness Theorem asserts that if a bounded, convex and closed subset C of a Banach space E satisfies that every $x^* \in E^*$ attains its supremum on C, then C is weakly compact. Motivated by a question posed by F. Delbaen, we will study cases in which we can deduce that C is weakly compact when a priori we know that only certain functionals of E^* (characterized by a geometrical property) attain their supremum on C.
This is a joint work with B. Cascales and J. Orihuela.

Weighted Bernstein inequalities

Sergey Tikhonov *Centre de Recerca Matemàtica*

In this talk I discuss the Bernstein inequalities with doubling and nondoubling weights

Projective norm of products of random gaussian matrices

Ignacio Villanueva *Universidad de Complutense de Madrid*

Motivated by a question in Quantum Information Theory we consider the following question: We let $(u_i)_{i=1}^n$, $(v_j)_{j=1}^n$ be m-dimensional normalized gaussian vectors. We consider the $n \times n$ matrix $\gamma = (\langle u_i, v_j \rangle)_{i,j=1}^n$ as an element of $\ell_\infty^n \otimes \ell_\infty^n$ and we study how likely is it that the projective norm of γ is strictly bigger than 1.
We show that the answer depends on the ratio $\alpha = \frac{m}{n}$: If $\alpha < 0.004$ then asymptotically almost every such γ has projective norm strictly bigger than 1, whereas if $\alpha > 2$ then asymptotically almost every such γ has projective norm strictly smaller than 1.
To prove this we use Grothendieck's inequality and tools from Random Matrix Theory. In particular, we need to calculate (with high probability) certain euclidean distance between a gaussian matrix and its Gram-Schmidt orthonormalization. The knowledge of this distance allows us to improve previous results in the area.
Joint work with C. E. González-Guillén, C. H. Jiménez, C. Palazuelos.