MULTILINEAR STABILITY OF CLASSES OF VECTOR-VALUED SEQUENCES

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Dedicated to Richard on the occasion of his 70th birthday.

This is a (part of a) joint work with Jamilson Campos (João Pessoa).

Motivation

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• Almost summing linear or multilinear operators send unconditionally summable sequences to almost unconditionally summable sequences.

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Usually the linearity of the operator plays an important role in the proof the linear stability.

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We call $X(\cdot)$ a sequence class.

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is a well-defined continuous n-linear operator and $\|\widehat{A}\| = \|A\|$.

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Linear stability $\not\Longrightarrow$ multilinear stability

Consider the bilinear operator

$$A\colon \ell_2 \times \ell_2 \longrightarrow \ell_1 \ , \ A\left((x_j)_{j=1}^{\infty}, (y_j)_{j=1}^{\infty}\right) = (x_j y_j)_{j=1}^{\infty}.$$

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This makes the multilinear stability of a given sequence class a typical multilinear problem.

Weakly and unconditionally summable sequences

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Using the trick: for every $k \in \mathbb{N}$, $\displaystyle\sum_{j=1}^{k} A(x_j^1, \ldots, x_j^n) =$

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it is not difficult to prove that, for $|\lambda_j^i| \leq 1$, $j = 1, \ldots, k$, $i = 1, \ldots, n$,

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$$\left\| \sum_{j=1}^{k} \lambda_j^1 \cdots \lambda_j^n A(x_j^1, \ldots, x_j^n) \right\| \leq \|A\| \cdot \prod_{m=1}^{n} \left\| (x_j^m)_{j=1}^{k} \right\|_{w,1} .$$

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Recalling that $\|(A(x_j^1, \ldots, x_j^n)_{j=1}^k\|_{w,1} =$

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Taking the supremum over k we get $(A(x_j^1,\ldots,x_j^n))_{j=1}^\infty \in \ell_1^w(F)$.

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The case of $\ell_p^u(\cdot)$ follows from the case of $\ell_p^w(\cdot)$.

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It is well known that $\mathrm{Rad}(E) \overset{1}{\subseteq} \mathrm{RAD}(E)$,

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It is well known that $\mathrm{Rad}(E) \overset{1}{\subseteq} \mathrm{RAD}(E)$, and

$$\mathrm{Rad}(E) = \mathrm{RAD}(E) \Longleftrightarrow c_0 \not\hookrightarrow E.$$

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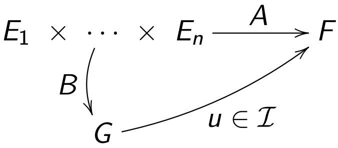

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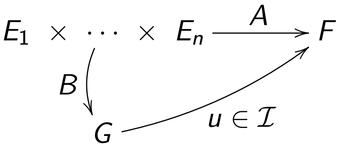

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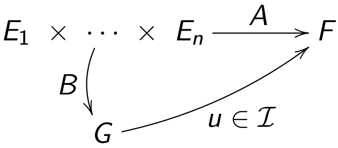

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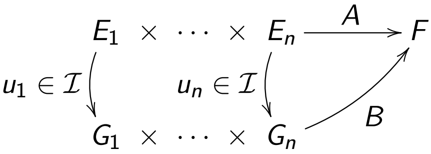

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Corollary. Any multilinear operator belonging to either $\pi_p^{\mathrm{dual}} \circ \mathcal{L}$ or $\mathcal{L} \circ \pi_p^{\mathrm{dual}}$ for some $p \geq 1$ is Cohen almost summing.

MUCHAS GRACIAS!